PROCEEDINGS

The 20th International and National Conference on Applied Computer Technology and Information Systems (ACTIS) and The International and National Conference on Business Administration (NCOBA) 2023-2

Journal & Proceeding ACTIS Applied Computer Technology and Information System

AiAT
ARTIFICIAL INTELLIGENCE ASSOCIATION OF THAILAND

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สารจากคณบดีคณะเทคโนโลยีสารสนเทศและนวัตกรรมดิจิทัล
มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าพระนครเหนือ

งานประชุมวิชาการระดับชาติและระดับนานาชาติด้านเทคโนโลยีคอมพิวเตอร์ประยุกต์และระบบสารสนเทศ ครั้งที่ 20 (ACTIS2023: The 20th International conference in Applied Computer Technology and Information System Acronym) และงานประชุมวิชาการระดับชาติและระดับนานาชาติ ด้านบริหารธุรกิจ ครั้งที่ 20 (NCOBA2023: The 20th National Conference on Business Administration) จัดขึ้นในวันที่ 25 สิงหาคม พ.ศ. 2566 โดยคณะเทคโนโลยีสารสนเทศและนวัตกรรมดิจิทัล มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าพระนครเหนือ คณะเทคโนโลยีดิจิทัล มหาวิทยาลัยราชพฤกษ์ สมาคมปัญญาประดิษฐ์ประเทศไทย และเครือข่ายความร่วมมือในการจัดงานประชุมวิชาการ จำนวน 11 สถาบันการศึกษา งานประชุมวิชาการนี้เป็นอีกหนึ่งเวทีในการแลกเปลี่ยนแนวความคิด องค์ความรู้ สร้างนวัตกรรมด้านเทคโนโลยีคอมพิวเตอร์ ระบบสารสนเทศประยุกต์ เทคโนโลยีดิจิทัล และบริหารธุรกิจ มีวัตถุประสงค์เพื่อร่วมสร้างความเข้มแข็งทางวิชาการในการจัดบริการวิชาการ การทำวิจัย และการพัฒนาบุคลากร เพื่อการพัฒนาประเทศชาติอย่างยั่งยืน

งานประชุมวิชาการครั้งนี้ต้องขอขอบคุณทุกท่านที่ได้ส่งบทความเพื่อเข้าสู่กระบวนการการพิจารณาซึ่งเป็นบทความที่มีคุณภาพสูงทำให้งานประชุมวิชาการได้รักษาคุณภาพและมาตรฐานของงาน ขอขอบพระคุณคณะกรรมการพิจารณาบทความทุกท่านที่เสียสละเวลาอันมีค่าเพื่อให้ได้ผลงานวิจัยที่มีคุณภาพได้นำเสนอในงานประชุมวิชาการในครั้งนี้ด้วย

ขอขอบคุณเครือข่ายความร่วมมือทางเทคโนโลยีคอมพิวเตอร์ประยุกต์และระบบสารสนเทศในประเทศทั้ง 11 สถาบันการศึกษา ที่ให้การสนับสนุนเป็นอย่างดีทั้งด้านคณาจารย์ บุคลากรในการร่วมเป็นคณะกรรมการเครือข่าย คณะกรรมการดำเนินงาน และคณะกรรมการพิจารณาบทความ ได้แก่ มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าพระนครเหนือ มหาวิทยาลัยเทคโนโลยีราชมงคลสุวรรณภูมิ มหาวิทยาลัยเทคโนโลยีราชมงคลธัญบุรี มหาวิทยาลัยเทคโนโลยีราชมงคลกรุงเทพ มหาวิทยาลัยราชภัฏจันทรเกษม มหาวิทยาลัยสุโขทัยธรรมาธิราช มหาวิทยาลัยทักษิณ มหาวิทยาลัยราชพฤกษ์ มหาวิทยาลัยกรุงเทพสุวรรณภูมิ วิทยาลัยเซาธ์อีสท์บางกอก และมหาวิทยาลัยราชภัฏนครปฐม

ท้ายนี้หวังเป็นอย่างยิ่งว่างานประชุมวิชาการ ACTIS และ NCOBA จะเป็นเวทีในการนำเสนอผลงานวิจัยและนำไปสู่การพัฒนาตนเองเพื่อให้เป็นนักวิจัยที่ดีและมีคุณภาพสูงยิ่ง ๆ ขึ้นไป ACTIS และ NCOBA จะเป็นแพลตฟอร์มสำหรับการแลกเปลี่ยนความรู้ในด้านเทคโนโลยีคอมพิวเตอร์ ระบบสารสนเทศประยุกต์ เทคโนโลยีดิจิทัล และบริหารธุรกิจ สร้างแรงบันดาลใจให้กับนักวิจัยเพื่อคิดค้นนวัตกรรมและพัฒนาองค์ความรู้ใหม่ ๆ เพื่อการพัฒนาชาติอย่างยั่งยืนตลอดไป

(ผู้ช่วยศาสตราจารย์ ดร.สุนันทา สดสี)
คณบดีคณะเทคโนโลยีสารสนเทศและนวัตกรรมดิจิทัล
มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าพระนครเหนือ

Message from Dean, Faculty of Information Technology and Digital Innovation, KMUTNB

The 20th International Conference in Applied Computer Technology and Information System Acronym (ACTIS2023) and the 20th National Conference on Business Administration (NCOBA2023), held on 25 August, 2023. ACTIS & NCOBA are organized by Faculty of Information Technology and Digital Innovation, King Mongkut's University of Technology North Bangkok, Faculty of Digital Technology, Rajapruk University, Artificial Intelligence Association of Thailand, and eleven academic collaborations.

The conferences provide a unique platform both for exchanging innovative research in Applied Computer Technology, Information Systems, and Business Administration which aim to enhance collaboration network, researches, and improve the sustainability of our nation.These conferences made huge possible with innovative and progressive contributions from the research community. Herein, special thanks are due to all technical committee members for their diligent consideration of all submissions and for maintaining and preserving the high standards for which ACTIS & NCOBA are justifiably renowned. Regrettably, as a consequence of our rigorous peer review process, we were unable to accept some articles for presentation.

As well as, a debt of gratitude is owed to the eleven co-operative universities for their support and the contributions of staff members by serving in the network committees, executive director committees, and technical program committees. These universities are King Mongkut's University of Technology North Bangkok (KMUTNB), Rajamangala University of Technology Suvarnabhumi (RUS), Rajamangala University of Technology Thanyaburi (RMUTT), Rajamangala University of Technology Krungthep (RMUTK), Chandrakasem Rajabhat University (CRU), Sukhothai

Assistant Professor Dr. Dr.-Ing. Sunantha Sodsee
Dean, Faculty of Information Technology and Digital Innovation
King Mongkut's University of Technology North Bangkok

สารจากคณบดีคณะเทคโนโลยีดิจิทัล
มหาวิทยาลัยราชพฤกษ์

การประชุมวิชาการในครั้งนี้ มีวัตถุประสงค์เพื่อเป็นเวทีระดับชาติ และนานาชาติ ให้นักวิชาการ นักวิจัย คณาจารย์ นิสิต และนักศึกษา ได้เผยแพร่ผลงาน ได้รับฟังและแลกเปลี่ยนองค์ความรู้ ผ่านบทความวิจัยและนวัตกรรม ซึ่งเป็นประโยชน์ ต่อการพัฒนาประเทศ เพื่อสร้างเครือข่ายความร่วมมือพัฒนาความก้าวหน้าทางวิชาการและวิจัยกับสถาบันการศึกษาต่าง ๆ

จากความตระหนักในภารกิจของสถาบันอุดมศึกษาที่ต้องวิจัยเพื่อสร้าง องค์ความรู้ใหม่ด้านเทคโนโลยีคอมพิวเตอร์และระบบสารสนเทศประยุกต์รวมถึงการบริหารธุรกิจอันจะส่งผลต่อการพัฒนาความเจริญก้าวหน้าของประเทศ คณะเทคโนโลยี สารสนเทศและนวัตกรรมดิจิทัล มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าพระนครเหนือ และคณะเทคโนโลยีดิจิทัล มหาวิทยาลัยราชพฤกษ์ ได้รับมอบหมายจากเครือข่ายเป็นเจ้าภาพร่วมจัดการประชุมวิชาการระดับชาติและนานาชาติ ด้านเทคโนโลยีคอมพิวเตอร์และระบบสารสนเทศประยุกต์ (ACTIS) การประชุมวิชาการระดับชาติและนานาชาติ ด้านบริหารธุรกิจ (NCOBA) ครั้งที่ 20 ซึ่งการประชุมวิชาการในครั้งนี้ได้รับความร่วมมือจากสมาคมปัญญาประดิษฐ์ ประเทศไทย

สุดท้ายนี้ ขอขอบคุณเครือข่ายการประชุมวิชาการ ACTIS และ NCOBA ทั้ง 11 มหาวิทยาลัย และคณะกรรมการจัดงานประชุมวิชาการจากคณะเทคโนโลยีสารสนเทศและนวัตกรรมดิจิทัล มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าพระนครเหนือ และคณะเทคโนโลยีดิจิทัล มหาวิทยาลัยราชพฤกษ์ ที่มีความมุ่งมั่นและความพยายาม เพื่อให้การประชุมวิชาการ ACTIS และ NCOBA ครั้งที่ 20 ปี 2023-2 นี้เป็นงานที่สร้างความประทับใจและน่าจดจำสำหรับผู้เข้าร่วมทุกท่าน

ดิฉันหวังไว้เป็นอย่างยิ่งว่าทุกท่านจะมีช่วงเวลาที่ดี และได้รับประโยชน์จากการเข้าร่วมการประชุมวิชาการ ACTIS และ NCOBA ครั้งที่ 20 ปี 2023-2 และหวังว่าทุกท่านจะเข้าร่วมการประชุมวิชาการ ACTIS และ NCOBA ครั้งที่ 21 ปี 2024 ต่อไป

ผู้ช่วยศาสตราจารย์ ดร.เรวดี ศักดิ์ดุลยธรรม
คณบดีคณะเทคโนโลยีดิจิทัล
มหาวิทยาลัยราชพฤกษ์

Message from the Dean, Faculty of Digital Technology, Rajapruk University

This conference is held with the purposes in being a stage for scholars, researchers, professors, students to present and disseminate their work for public engagement and knowledge exchange through research articles and innovation which will be beneficial for country development and to build a cooperation network on the development of academic achievement and research with other educational institutions.

Due to the awareness of a mission of higher education institutions in conducting a research study to build a new knowledge in the field of applied computer technology and information system including business administration, which shall affect the development and progress of the country, Faculty of Information Technology and Digital Innovation, King Mongkut's University of Technology North Bangkok and Digital Technology Faculty, Rajapruk University were assigned from the Network to be the co-host for holding the 20th International and National Conference on Applied Computer Technology and Information Systems (ACTIS) and the International and National Conference on Business Administration (NCOBA). This academic conference received good cooperation from Artificial Intelligence Association of Thailand (AIAT).

Finally, I thank the network of the International and National Conference on ACTIS and NCOBA from all 11 universities and the conference joint-organizing committees from Faculty of Information Technology and Digital Innovation, King Mongkut's University of Technology North Bangkok and Digital Technology Faculty, Rajapruk University who have shown great commitment and fantastic team efforts in ensuring that the 20th ACTIS and NCOBA 2023-2 will be an impressive and memorable event for all participants.

I wish all of you a delightful and fruitful time in the 20th ACTIS and NCOBA 2023-2 and I look forward to your participation in the 21th ACTIS and NCOBA 2024.

Raywadee

Assistant Professor Dr. Raywadee Sakdulyatham
Dean, Faculty of Digital Technology
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1.4 Storage Systems and Techniques
1.5 Data Retrieval & Data Mining 1.6 System Modeling and Simulations

Commerce Technology

2.1 e-Business Applications and Software
2.2 Managing Innovation & Marketing on the Web
2.3 Web Advertising and Web Publishing
2.4 Business & Consumer Oriented E-Commerce
2.5 Business Technology

Software Engineering

3.1 Software Process, Design and Architecture
3.2 Software Configuration Management
3.3 Software Evolution
3.4 Software Component and Prototyping

Computer Systems

4.1 Computer Systems and Applications
4.2 Distributed & Grid Computing
4.3 Computer Graphics & HCI
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4.5 Network Management

Computer Education & Training
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NCOBA – Conference Tracks:

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1.1 Principle of International Business
1.2 International Business Management
1.3 Business Development
1.4 Innovation and Technology Management

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2.1 Principle of Business Management
2.2 Marketing Environment Analysis
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Finance

3.1 Banking and Finance
3.2 Corporate Finance and Governance
3.3 International Finance

Economics

4.1 Economic Development
4.2 Economic Methodology
4.3 Labour Economics
4.4 International Economics

Table of Articles

The International Conference on Applied Computer Technology and Information Systems (ACTIS)

Paper ID	Title	Researcher Name	Page
ACTIS 2023-5	Leveraging Three Image Processing Techniques and Machine Learning for Milled Rice Variety Classification	Kanchanok Udomjetjamnong, Piyanart Boonramart and Jessada Tanthanuch	56
ACTIS 2023-8	A Comparative Study between Generalized Linear Models and Generalized Additive Models in the Modeling of Health Biological Signal Data	Natakon Nawaratana, Amornrat Suriyawichitseranee, and Jessada Tanthanuch	61
ACTIS 2023-9	Price Prediction of Bitcoin Based on Automatic Features Engineering and Machine Learning Techniques	Phetngam Koatborom and Benjawan Rodjanadid	68
ACTIS 2023-12	Stock Closing Price Prediction Using Feature Engineering and Machine Learning Techniques	Ratchapon Pariyothai, Jirakit Boonmunewai and Benjawan Rodjanadid	77
ACTIS 2023-14	Using RFM and K-means for Customer Segmentation on AI service platform	Panumas Sitthikarn and Ekarat Rattagan	85
ACTIS 2023-18	The Algorithm to Determine the Number of Cameras Placed for Roadway Monitoring	Amphon Kliaram and Akanat Wetayawanich	91
ACTIS 2023-19	Utilizing Bayesian Analysis of Wrapped Distributions in Computer Technology	Mangkon Damnet, Amornrat Suriyawichitseranee and Jessada Tanthanuch	97
ACTIS 2023-25	A Risk Area Notifications on Mobile: A Case Study of Three Southern Border Provinces in Thailand	Suwimol Jungjit, Phaklen Ehkan and Amonrat Prasitsupparote	102

Utilizing Bayesian Analysis of Wrapped Distributions in Computer Technology

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***Abstract*—Conventional distributions primarily represent one-dimensional distributions that can take on any real value. In contrast, wrapped distributions are specifically tailored to handle data exhibiting circular or periodic characteristics. The wrapped distribution is purposefully crafted to accommodate a random variable drawn from a conventional distribution and map it onto the unit circle. It finds applications in various facets of daily life, including finance, weather forecasting, medicine, sports, gaming, and computer applications. This research delves into the notion of extending the application of Bayesian analysis to wrapped distributions. The study underscores the key distinctions between Bayesian analysis for conventional distributions and the application of Bayesian analysis to related wrapped distributions. Additionally, the study provides insights into applications within the realm of computer technology.**

Keywords—Wrapped Distribution, Bayesian Analysis, Bayes' theorem, Circular data.

I. Introduction

Conventionally, the *distribution function* or *cumulative distribution function* is defined for the *random variable X*, which is a finite single-valued function mapping to real value . There are some types of data defined on the circumference of a unit circle, called *circular random variables*. The range of a circular random variable are usually measured in radians, in the range [0,2π) or [-π, π], or in degrees. However, there exists another type of random variable where all possible values are confined to the interval between 0 and 2π (or between – π and π), forming a wrapped random variable. Wrapped random variables are generally simpler to handle mathematically compared to circular random variables. This is because wrapped random variables can be represented as numbers, whereas circular random variables may encompass values that are not necessarily numerical. In various fields such as meteorology, oceanography, and geology, circular random variables are extensively utilized to examine cyclic phenomena like wind patterns, ocean currents, tides, and geological orientations. On the other hand, wrapped random variables are employed in studies involving the direction of a river, once the direction has been confined to the unit circle. Additionally, they are used to analyze the angle of a slope, after the angle has been confined within the unit circle. The probability distributions that describe the possible values of the wrapped random variable are called *wrapped distributions* [1]. Wrapped distributions find applications in various aspects of daily life, encompassing fields such as finance, weather forecasting, medicine, sports, gaming, and computer applications [2,3].

Considering the central role of probability distributions in Bayesian analysis, which is a statistical approach that utilizes Bayes' theorem to revise our beliefs regarding a hypothesis as we gain additional information. Essentially, it enables us to integrate our initial understanding of a subject into the analysis of new data. Bayesian analysis enables the representation of uncertainty linked to the parameters of interest. A particularly powerful aspect is employing Bayes' theorem to update the prior distribution to the posterior distribution. The prior distribution is a probability distribution that expresses our beliefs about the hypothesis prior to observing the data. It may be constructed based on our knowledge of the subject matter or selected subjectively. On the other hand, the posterior distribution is a probability distribution that captures our beliefs about the hypothesis after analyzing the data [4].

Exploring the application of Bayesian analysis to wrapped distributions becomes highly intriguing. This research primarily concentrates on utilizing Bayesian analysis to identify conjugate prior distributions for wrapped distributions. Illustrations of conjugate prior distributions are provided as examples in the field of computer technology.

II. Related Statistical Distributions

A. Wrapped Distribution

A linear random variable X on the real line can be transformed into a circular random variable by applying a modulo 2π reduction, $\Theta = X(\bmod 2\pi)$. The operation wraps the linear random variable around a circle with a unit radius. The probability is then accumulated over all the overlapping points $x = \theta, \theta \pm 2\pi, \theta \pm 4\pi, \ldots$. Therefore,

$$f_\Theta(\theta) = \sum_{m=-\infty}^{\infty} f_X(\theta + 2\pi m), \quad 0 \le \theta < 2\pi,$$

where $f_\Theta(\theta)$ represents the circular density and $f_X(x)$ denotes the density of real-valued random variable. It is important to note that the presented distribution is applicable to both discrete and continuous wrapped distribution [1].

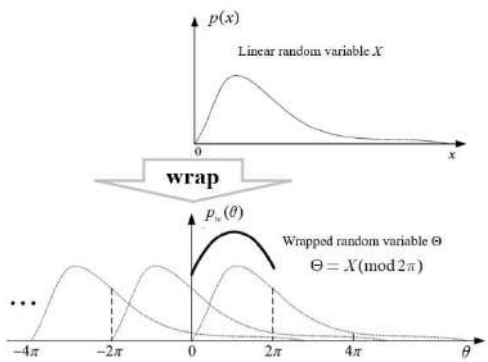

Fig. 1. Illustration of obtaining a wrapped distribution by wrapping the linear random variable (adapted from images in [5]).

B. Bayes' Theorem

Let $P(A)$ and $P(A|B)$ denote the probability of A and the probability of A occurring given that B is true, respectively.

Theorem (Bayes).

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)} = \frac{P(B \mid A) \cdot P(A)}{\int_{\alpha} P(B \mid A_{\alpha}) \cdot P(A_{\alpha}) dA_{\alpha}},$$

where A and B are events and $P(B) \neq 0$. $P(A|B)$ is called the *posterior probability* of A given B and $P(B|A)$ is interpreted as the *likelihood* of A given a fixed B. $P(A)$ and $P(B)$ are referred to as the *prior probability* and *marginal probability*, respectively [6].

C. Bayesian Analysis

Bayesian analysis stands as a powerful tool for deriving inferences about unknown parameters within a statistical model. It relies on Bayes' theorem, which facilitates the refinement of our beliefs concerning a parameter based on emerging evidence. Bayesian analysis offers the capability to furnish a more comprehensive understanding of the uncertainties inherent in our inferences.

Consider a random variable X, which has a probability distribution dependent on the symbol λ, where λ belongs to a well- defined set Ω. Previously, λ was regarded as an unknown parameter. Now, let us introduce a new random variable Λ that has a probability distribution over the set Ω. Just as x is considered as a possible value of the random variable X, λ is now considered as a potential value of the random variable Λ. Hence, the distribution of X is influenced by λ, which serves as an experimental value of the random variable Λ. The probability density function (pdf) of Λ is denoted as $h(\lambda)$, and when λ is not an element of Ω, $h(\lambda)$ is set to zero. This pdf $h(\lambda)$ is referred to as the *prior probability density function* (prior pdf) of Λ. The distribution defined by this prior pdf is called the *prior distribution.* Additionally, the pdf of X is denoted as $f(x \mid \lambda)$ since it is viewed as a *posterior probability density function* (posterior pdf) of X, given $\Lambda = \lambda$. Note that posterior pdf is also called the *conditional probability density function*

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the conditional distribution of X given $\Lambda=\lambda$ with pdf $f(x \mid \lambda)$. Let $\mathbf{X}'=(X_1, \ldots, X_n)$ and $\mathbf{x}'=(x_1, \ldots, x_n)$ denote the transposed vector notations of variables. Therefore, the joint conditional pdf of $\mathbf{X}$, given $\Lambda= \lambda$, is

$$L(\mathbf{x} \mid \lambda)= f(x_1 \mid \lambda) f(x_2 \mid \lambda) \cdots f(x_n \mid \lambda).$$

The joint pdf of $\mathbf{X}$ and Λ is

$$g(\mathbf{x}, \lambda) = L(\mathbf{x} \mid \lambda) h(\lambda).$$

If Λ is a random variable of the continuous type, the joint marginal pdf of $\mathbf{X}$ is given by

$$\bar{g}(\mathbf{x}) = \int_{-\infty}^{\infty} g(\mathbf{x}, \lambda) d\lambda.$$

If Λ is a random variable of the discrete type, the summation replaces the integration. The posterior pdf of Λ, given by the sample $\mathbf{X}$, is

$$k(\lambda \mid \mathbf{x}) = \frac{g(\mathbf{x}, \lambda)}{\bar{g}(\mathbf{x})}.$$

The distribution characterized by this posterior pdf is known as the *posterior distribution*. The prior distribution represents the subjective belief regarding Λ before the sample is obtained, whereas the posterior distribution is the conditional distribution of Λ after the sample is obtained.

D. The Gamma Distribution

The gamma distribution is a continuous probability distribution. This distribution finds application in scenarios where the focus is on measuring the time between events. For instance, it can be used to represent the time intervals between customer arrivals at a store or the time intervals between failures of a machine. The pdf of gamma distribution is given by:

$$h(x|\alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x},$$

where α is a shape parameter and β is a scale parameter. The pdf of the gamma distribution exhibits a right- skew pattern, with the extent of skewness being controlled by the shape parameter. A higher shape parameter will result in a distribution with a longer tail, whereas a lower shape parameter will result in a distribution with a shorter tail. A higher scale parameter will result in a distribution encompassing a wider range of values, whereas a lower scale parameter will result in a distribution encompassing a narrower range of values.

E. Hurwitz Zeta Function

The Hurwitz Zeta function can be seen as an extension of the Riemann Zeta function. It is named after the Austrian mathematician Adolf Hurwitz, who introduced it in the late 19th century. Its definition is as follows:

$$\zeta(s, a) = \sum_{n=1}^{\infty} \frac{1}{(n+a)^s},$$

where a is a real number and s is a complex number with real part greater than 1 [7].

III. The Wrapped Distribution of Exponential Distribution

A. The Exponential Distribution

The exponential distribution is a continuous probability distribution that describes the time between events in a Poisson process, where events occur continuously and independently at a constant average rate. It is characterized by its rate parameter $\lambda > 0$, which is the reciprocal of the average time between events. The distribution of a random variable x is Exponential if it has probability density function

$$f(x \mid \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases},$$

where X is a linear random variable [8]. The pdf of the exponential distribution is a monotonically decreasing function. Consequently, as the time between events increases, the probability of a longer duration between events diminishes.

B. The Wrapped Distribution of Exponential Distribution

The wrapped exponential distribution is a probability distribution that arises when modeling cyclic or periodic phenomena. It is derived from the exponential distribution by wrapping it around a circular or periodic domain. The distribution of a random variable x is wrapped exponential if it has probability density function

$$\begin{aligned} f_\Theta(\theta|\lambda) &= \sum_{k=-\infty}^{\infty} f_X(\theta + 2\pi k|\lambda) \\ &= \sum_{k=0}^{\infty} \lambda e^{-\lambda(\theta+2\pi k)} \\ &= \frac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}, \end{aligned}$$

where Θ is a wrapped random variable and λ is a parameter [9]. The pdf of the wrapped exponential distribution is a decreasing function. Consequently, as the angle increases, the likelihood of observing a larger angle diminishes.

IV. Bayesian Analysis of Wrapped Exponential Distribution

A. Uninformative Prior

If we assume that our observed data occurs uniformly, it is commonly assumed that the prior distribution should also be uniform, $h(x) = 1$. The Bayesian analysis of the wrapped exponential distribution with the mentioned prior distribution is presented as follows:

$$\begin{aligned} & g(\lambda|\theta) \\ &= \frac{f_\Theta(\theta \mid \lambda) \cdot h(\lambda)}{\int_0^\infty f_\Theta(\theta \mid \lambda) \cdot h(\lambda)\, d\lambda} \\ &= \frac{\sum_{k=0}^{\infty} \lambda e^{-\lambda(\theta+2\pi k)}}{\int_0^\infty \sum_{k=0}^{\infty} \lambda e^{-\lambda(\theta+2\pi k)}\, d\lambda} \\ &= \frac{\dfrac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}}{\sum_{k=0}^{\infty}\left[\left(\int_0^\infty \dfrac{(\theta+2\pi k)^2}{\Gamma(2)} \lambda^{2-1} e^{-\lambda(\theta+2\pi k)}\, d\lambda\right)\dfrac{\Gamma(2)}{(\theta+2\pi k)^2}\right]} \\ &= \frac{\dfrac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}}{\sum_{k=0}^{\infty}\left[\left(\int_0^\infty \text{Gamma}\left(\lambda|2, \theta+2\pi k\right) d\lambda\right)\dfrac{\Gamma(2)}{(\theta+2\pi k)^2}\right]} \\ &= \frac{\dfrac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}}{\sum_{k=0}^{\infty} \dfrac{1}{\left(\theta+2\pi k\right)^2}} \\ &= \frac{\dfrac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}}{\dfrac{1}{(2\pi)^2}\sum_{k=0}^{\infty} \dfrac{1}{\left(k+\dfrac{\theta}{2\pi}\right)^2}} \\ &= \frac{4\pi^2 \dfrac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}}{\sum_{k=0}^{\infty} \dfrac{1}{\left(k+\dfrac{\theta}{2\pi}\right)^2}}, \end{aligned}$$

where $\Gamma(\cdot)$ is the gamma function and $\text{Gamma}(\cdot|\cdot)$ is the gamma distribution.

Since

$$\sum_{k=0}^{\infty} \frac{1}{\left(k+\dfrac{\theta}{2\pi}\right)^2}$$

can be represented by the Hurwitz Zeta Function, then

$$g(\lambda|\theta) = \frac{4\pi^2 \lambda e^{-\lambda\theta}}{\left(1-e^{-2\pi\lambda}\right)\zeta\left(2, \dfrac{\theta}{2\pi}\right)},$$

where $\zeta(\cdot,\cdot)$ is The Hurwitz Zeta Function.

B. Gamma Prior Distribution

If we assume that our prior distribution is gamma type,

$$h(x)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}.$$

The following presents the Bayesian analysis of the wrapped exponential distribution, considering the gamma prior distribution:

$$g(\lambda|\theta)$$

$$=\frac{f_{\Theta}(\theta\,|\,\lambda)\cdot h(\lambda)}{\int_0^{\infty} f_{\Theta}(\theta\,|\,\lambda)\cdot h(\lambda)\,d\lambda}$$

$$=\frac{\left(\sum_{k=0}^{\infty}\lambda e^{-\lambda(\theta+2\pi k)}\right)\left(\frac{\beta^{\alpha}}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\lambda\beta}\right)}{\int_0^{\infty}\left(\sum_{k=0}^{\infty}\lambda e^{-\lambda(\theta+2\pi k)}\right)\left(\frac{\beta^{\alpha}}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\lambda\beta}\right)d\lambda}$$

$$=\frac{\left(\frac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}\right)\left(\frac{\beta^{\alpha}}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\lambda\beta}\right)}{\frac{\beta^{\alpha}}{\Gamma(\alpha)}\int_0^{\infty}\sum_{k=0}^{\infty}\lambda^{(\alpha+1)-1}e^{-\lambda(\beta+\theta+2\pi k)}\,d\lambda}$$

$$=\frac{\frac{\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{1-e^{-2\pi\lambda}}}{\sum_{k=0}^{\infty}\left[\left(\int_0^{\infty}\frac{(\beta+\theta+2\pi k)^{\alpha+1}}{\Gamma(\alpha+1)}\lambda^{(\alpha+1)-1}e^{-\lambda(\beta+\theta+2\pi k)}d\lambda\right)\frac{\Gamma(\alpha+1)}{(\beta+\theta+2\pi k)^{\alpha+1}}\right]}$$

$$=\frac{\frac{\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{1-e^{-2\pi\lambda}}}{\sum_{k=0}^{\infty}\left[\left(\int_0^{\infty}\text{Gamma}\left(\lambda\,|\,\alpha+1,\beta+\theta+2\pi k\right)d\lambda\right)\frac{\Gamma(\alpha+1)}{(\beta+\theta+2\pi k)^{\alpha+1}}\right]}$$

$$=\frac{\frac{\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{1-e^{-2\pi\lambda}}}{\Gamma(\alpha+1)\sum_{k=0}^{\infty}\frac{1}{(\beta+\theta+2\pi k)^{\alpha+1}}}$$

$$=\frac{\frac{\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{1-e^{-2\pi\lambda}}}{\frac{\Gamma(\alpha+1)}{(2\pi)^{\alpha+1}}\sum_{k=0}^{\infty}\frac{1}{\left(k+\frac{\theta+\beta}{2\pi}\right)^{\alpha+1}}}$$

$$=\frac{\frac{\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{1-e^{-2\pi\lambda}}}{\frac{\Gamma(\alpha+1)}{(2\pi)^{\alpha+1}}\zeta\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)}$$

$$=\frac{(2\pi)^{\alpha+1}\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{\Gamma(\alpha+1)\left(1-e^{-2\pi\lambda}\right)\zeta\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)},$$

where $\Gamma(\cdot)$ is the gamma function, $\text{Gamma}(\cdot\,|\,\cdot)$ is the gamma distribution, and $\zeta(\cdot,\cdot)$ is the Hurwitz Zeta function.

V. Posterior Probability of Wrapped Exponential Distribution

A. The Posterior Distribution with Uninformative Prior

For the wrapped exponential distribution, assuming that the observed data occurs uniformly, the pdf of the posterior distribution is as follows:

$$g(\lambda|\theta)=\frac{4\pi^2\lambda e^{-\lambda\theta}}{\left(1-e^{-2\pi\lambda}\right)\zeta\left(2,\frac{\theta}{2\pi}\right)},$$

where Θ is the random variable of the wrapped exponential distribution representing the time between events, λ is the rate parameter of the exponential distribution determining the average number of events per unit of time, and $\zeta(\cdot,\cdot)$ is the Hurwitz Zeta function.

B. The Posterior Distribution with Gamma Prior Distribution

The posterior distribution for the wrapped exponential distribution with a gamma prior distribution as the pdf given by the following:

$$g(\lambda|\theta)=\frac{(2\pi)^{\alpha+1}\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{\Gamma(\alpha+1)\left(1-e^{-2\pi\lambda}\right)\zeta\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)},$$

where Θ, λ, and $\zeta(\cdot,\cdot)$ are the parameters defined in the previous section.

C. Some Properties of Obtained Posterior Wrapped Disitributions

The comparison of distributions obtained is summarized, as shown in Table I. Some properties and parameters of the distribution that have been analyzed are presented in Table II.

TABLE I. Obtained Posterior Wrapped Disitributions

Feature	The Posterior Distribution with Uninformative Prior	The Posterior Distribution with Gamma Prior Distribution
Prior Information	Uninformative	Gamma Distribution
Prior	$h(x)=1$	$h(x)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$
Posterior	$g(\lambda\|\theta)=\frac{4\pi^2\lambda e^{-\lambda\theta}}{(1-e^{-2\pi\lambda})\zeta\left(2,\frac{\theta}{2\pi}\right)}$	$g(\lambda\|\theta)=\frac{(2\pi)^{\alpha+1}\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{\Gamma(\alpha+1)(1-e^{-2\pi\lambda})\zeta\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)}$

TABLE II. Some Properties of Obtained Posterior Wrapped Disitributions

Feature	The Posterior Distribution with Uninformative Prior	The Posterior Distribution with Gamma Prior Distribution
Support	$\lambda>0$	$\lambda>0$
Parameter	$\theta\in(0,2\pi]$	$\theta\in(0,2\pi],\alpha,\beta>0$
PDF	$\frac{4\pi^2\lambda e^{-\lambda\theta}}{(1-e^{-2\pi\lambda})\zeta\left(2,\frac{\theta}{2\pi}\right)}$	$\frac{(2\pi)^{\alpha+1}\lambda^{\alpha}e^{-\lambda(\theta+\beta)}}{\Gamma(\alpha+1)(1-e^{-2\pi\lambda})\zeta\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)}$

Feature	The Posterior Distribution with Uninformative Prior	The Posterior Distribution with Gamma Prior Distribution
MGF	$\frac{\zeta\left(2,\frac{\theta-t}{2\pi}\right)}{\zeta\left(2,\frac{\theta}{2\pi}\right)}, t<\theta$	$\frac{\zeta\left(\alpha+1,\frac{\theta+\beta-t}{2\pi}\right)}{\zeta\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)}, t<\theta+\beta$
Mean	$\frac{2\zeta\left(3,\frac{\theta}{2\pi}\right)}{2\pi\zeta\left(2,\frac{\theta}{2\pi}\right)}$	$\frac{(\alpha+1)\zeta(\alpha+2,\frac{\theta+\beta}{2\pi})}{2\pi\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)}$
Variance	$\frac{6\zeta\left(4,\frac{\theta}{2\pi}\right)}{(2\pi)^2\zeta\left(2,\frac{\theta}{2\pi}\right)} - \left(\frac{2\zeta\left(3,\frac{\theta}{2\pi}\right)}{2\pi\zeta\left(2,\frac{\theta}{2\pi}\right)}\right)^2$	$\frac{(\alpha+1)(\alpha+2)\zeta\left(\alpha+2,\frac{\theta+\beta}{2\pi}\right)}{(2\pi)^2\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)} - \left[\frac{(\alpha+1)\zeta(\alpha+2,\frac{\theta+\beta}{2\pi})}{2\pi\left(\alpha+1,\frac{\theta+\beta}{2\pi}\right)}\right]^2$

VI. Some Examples Explained By Posterior Wrapped Exponential Distribution

A. Uninformative Prior

If the prior distribution lacks information, it is reasonable to assume that the random variable follows a uniform occurrence. Let us consider the specific scenario of the failure of hard disk drives (HDDs). The Mean Time to Failure (MTTF) indicated in the datasheets of HDDs implies a consistent rate of failure, where the probability of failure density follows an exponential distribution. Nonetheless, real-world observations of HDD replacements show that the failure rate doesn't remain constant as the drive ages. This variance could be attributed to the intricate nature of HDD structures and the underlying modes of failure they encompass [10]. To ensure a uniform decay process for each HDD, we assume that $h(x) = 1$. For individuals working, a computer is turned on and off regularly. In this context, it might be more appropriate to utilize the wrapped exponential distribution with $h(x) = 1$ for modeling HDD failure scenarios, rather than relying on the linear exponential distribution. The wrapped exponential distribution takes into account the periodic nature of HDD decay, which arises as individuals regularly power their computers on and off.

B. The Posterior Distribution with Gamma Prior Distribution

When working within the computer system design, the task of queueing network analysis for an interactive computing system presents specific challenges. Pertinent information reveals that there exists a service time of t seconds between requirements, although the service rate within these intervals remains uncertain. To tackle this uncertainty, a combination of a gamma distribution preceding an exponential distribution can be employed [11]. This synergistic approach enables the creation of an extensive array of potential values for the service rate through the gamma distribution, while the exponential distribution furnishes a dependable probability distribution for the time span between requirements.

Additionally, it has been observed that user jobs often require another round when the deadline for the new round approaches. In such scenarios, the service time can be modeled using a wrapped exponential distribution, with a gamma distribution serving as the prior distribution.

Equipped with this valuable knowledge, a computer system designer can configure network equipment that strikes a fair and harmonious balance, taking into account the interests of both the company and its customers.

VII. Conclusion

This research has introduced the expansion of the conventional posterior statistical distribution concept into posterior wrapped statistical distributions, which are suitable for data that can be transformed from a linear format to confine within a unit circle. It has been found that this concept aligns with various scenarios within the realms of information technology and data science. The obtained posterior wrapped statistical distributions are appropriate for various phenomena occurring within these domains allows for more efficient prediction of behaviors, strategic management planning, and effective execution of related tasks. However, the research must also extend to other statistical distributions beyond exponential distributions in order to encompass applications of statistics in various other fields in the future.

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For The Paper Titled

Utilizing Bayesian Analysis of Wrapped Distributions in Computer Technology

25 August 2023

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