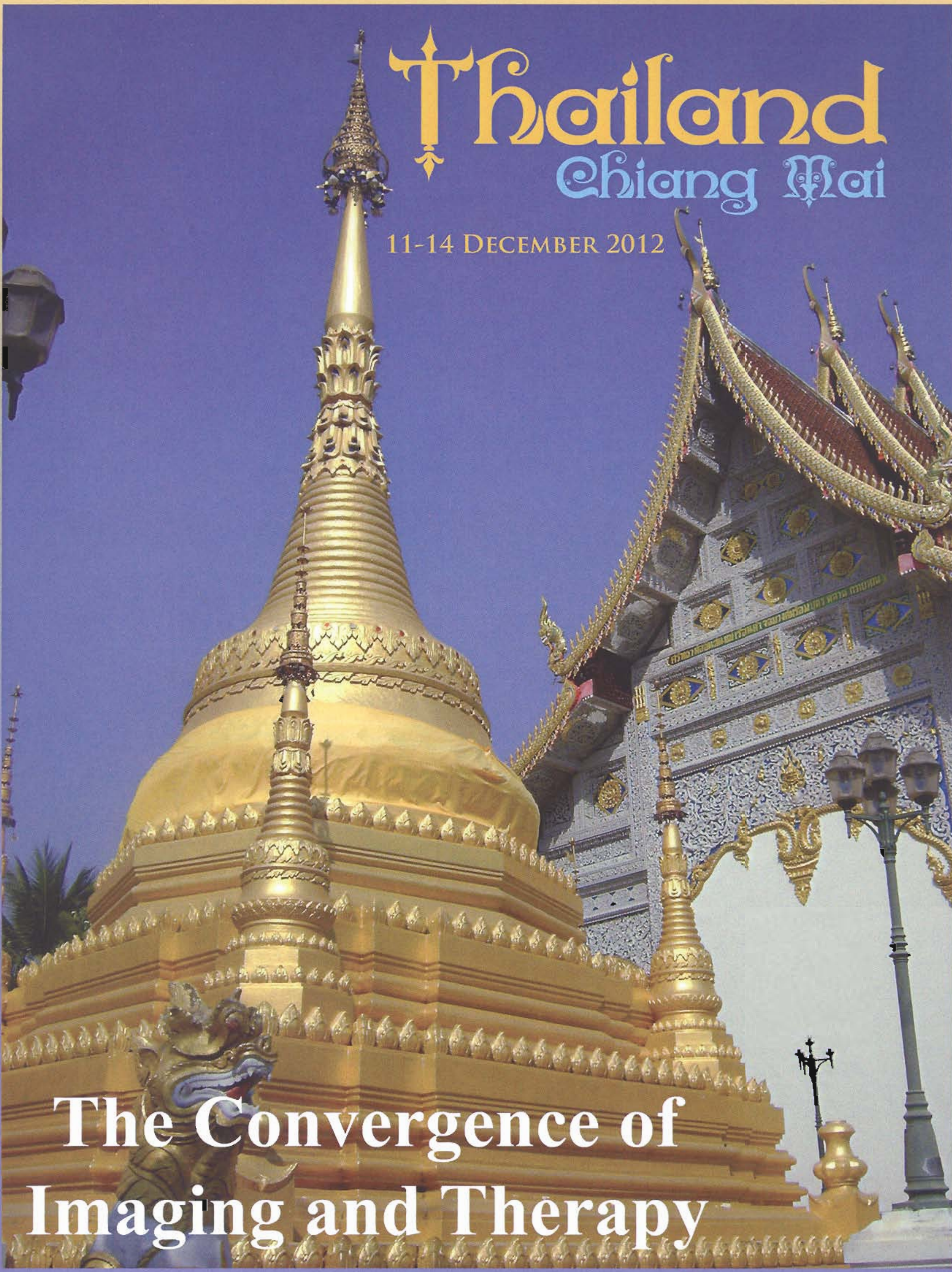

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The Convergence of Imaging and Therapy

PROCEEDINGS

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ULTRASOUND IMAGE ENHANCEMENT BY MEANS OF A VARIATIONAL APPROACH

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Abstract— **A mathematical model for images with speckle noise has been developed. To verify the model, sample images were first overlaid with noise and then de-noised by the obtained process. The reconstructed images were analyzed.**

Development of the model started with the observation that speckle noise is related with the Rayleigh distribution. Hence the image intensity can be described by a Rayleigh random variable. The maximum probability of the observed images on the condition of the noiseless image was required. This led to the formulation of a discrete model, which can be considered as the approximation of an integral model. By the variational approach, the problem of minimizing the integral model was transformed to a simpler one, i.e. finding the solution of an Euler-Langrange equation. In theory, the solution of the equation should provide the noiseless reconstructed image

For the purpose of verification, a pattern image and the Lenna image were used as sample images. Speckle noise with variance 0.02 was added to the original images by MATLAB software. The correlation coefficients of the original images and the noisy images were compared with the correlation coefficients of the original images and the reconstructed images. For the pattern image, the correlation coefficient of the original image and the noisy image was 0.9678 and the correlation coefficient of the original image and the reconstructed images with 200, 250, 300 and 350 iterative loops were 0.9963, 0.9974, 0.9980 and 0.9982 respectively. For the Lenna image, the correlation coefficient of the original image and the noisy image was 0.9444 and the correlation coefficient of the original image and the reconstructed images with 200, 250, 300 and 350 iterative loops were 0.9730, 0.9804, 0.9848 and 0.9871 respectively.

The results show that the model can used to remove noise from an ultrasound image.

Keywords— **calculus of variations, Rayleigh distribution, speckle noise, gradient descent method, ultrasound image**

I. INTRODUCTION

Ultrasound images provide low cost, non-invasive and real-time images which can help clinicians in diagnosis and therapy. However, the ultrasonic wave encounters rough surfaces which results in scattering and leads to noise speckle noise. Thus denoising model are an important topic in image processing research. [1] shows the comparison of speckle filters in radar images dealing with the minimum mean square error model. The main disadvantage of those filters is that we have to know the information of the noise a-priori in the computation. Difficulties arise when we work with an ultrasound video because we do not have the speckle noise information.

This problem can be solved by a mathematical model called the *variational approach*. The representation of the image in several variational models is given by the additive noise model, $u(x,y) = u(x,y) + n(x,y)$, or the multiplicative noise model, $u(x,y) = n(x,y)u(x,y)$, where $u(x,y)$ is the intensity of the desired image at coordinate (x,y), $u(x,y)$ is the intensity of the observed image at coordinate (x,y) and $n(x,y)$ is the intensity of noise at coordinate (x,y). The variational approach is used for finding the desired image u from the noisy image u.

There are several studies of digital image denoising models dealing with the variational approach, for example :

1. **The ROF model [2]**

In 1992, Rudin, Osher, and Fatemi presented a mathematical denoising model called the ROF model, which uses the additive noise model and is based on *calculus of variations*.

2. **The variatonal approach for Poisson noise [3]**

In 2007, Le, Chatrand and Asaki adapted the ROF model to present the data-fidelity term of the model which is suitable for Poisson noise.

3. **A variational approach to remove multiplicative noise [4]**

In 2008, Aubert and Aujol focused on the problem of multiplicative speckle noise removal.

Green [5] presented statistical description of the ROF model [2] which deals with the Gaussian noise. On the other hand, Le et al. [3] draw their inspiration from the modeling of Poisson noise. However a model for speckle noise reduction in ultrasound images is still needed.

In this research, a variational approach adapted from the ROF model is used to construct a model to reduce the speckle noise in the ultrasound image. Different from [3,4], we describe the image intensity of the ultrasound image by Rayleigh distribution and apply the variational approach to the additive noise model.

The model takes the form of an integral, and calculus of variations leads to the problem of an Euler-Lagrange partial differential equation. The solution of this equation is approximated by the *gradient descent method*. The pattern image and the Lenna image are used to evaluate the proposed model by comparing the *correlation coefficient* of the noisy images and the reconstructed images to the original ones. It is found that the model can be used to denoise noisy images and ultrasound videos. The description of the results is shown in this article.

II. Mathematical background and methods

A. Speckle Noise and Rayleigh Distribution

The *Rayleigh distribution* is a continuous probability distribution. It arises when a two-dimensional vector has elements that are *normally distributed* random variables, are *independent* and *both have zero mean* and *equal variance*. The vector's magnitude will then have a Rayleigh distribution [6]. It density function is known as the *Rayleigh density function* $R_\sigma(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$, where

$$\mathcal{R}_\sigma(r) = \begin{cases} 0 & \text{if } r < 0 \\ \int_0^r \frac{\tau}{\sigma^2} e^{-\frac{\tau^2}{2\sigma^2}} d\tau & \text{if } r \geq 0 \end{cases}$$

is the corresponding *Rayleigh distribution*, and σ is its variance.

Speckle is a random pattern which has a negative impact on *coherent imaging*, including ultrasound imaging. It is the result of the superposition of many waves, which have different or incoherent phases [7]. Speckle occurs in an ultrasound image because the ultrasonic wave encounters rough surfaces that result in the scattering of waves, each scattered wave from a rough surface has a different phase which leads to the forming of speckles. The amplitude of the harmonic wave corresponding to the intensity of the image at each point has a Rayleigh distribution. Here the amplitude is considered as a Rayleigh random variable.

B. Calculus of Variations

Calculus of Variations is a field of mathematics that deals with functionals. Such functionals can be formed as integrals involving an unknown function and its derivatives. The interest is in extremal functions making the functional attain an extremum value. For example, the problem involves finding the extrema of integrals of the form

$$\mathcal{I} = \iint_\Omega F(x, y, u, u_x, u_y)\, dxdy$$

over a bounded region Ω, where F is uniformly continuous on Ω. By calculus of variations, the solution of this problem is equivalent to the solution of the equation

$$\frac{\partial}{\partial x} F_{u_x} + \frac{\partial}{\partial y} F_{u_y} - F_u = 0,$$

which is called the *Euler-Lagrange differential equation.*

C. Description of the Proposed Model

Assume that u is a given noisy ultrasound image defined on Ω, a bounded open rectangle in $\mathbb{R}^2$ with piecewise *Lipschitz boundary* $\partial\Omega$. We assume u is bounded and positive on Ω and $u_{x,y}$ is the intensity of u at the location (x, y). Note that u is assumed to be noiseless on $\partial\Omega$. Let $U_{x,y}$ be a random variable on the set of noiseless images, which corresponds to noiseless image intensity at point (x, y) and $U_{x,y}$ be a random variable on the set of observed images, which corresponds to observed image intensity at point (x, y). We wish to determine the image u which is most likely to the given observed image u.

From the statistical point of view, we are going to find an image u which maximizes the conditional probability that the intensity of a noiseless image is most likely to the intensity of the given observed image for all (x, y) We assume that Ω is pixelated by $\Omega = \{(x, y) \mid x, y = 0, \ldots, N-1\}$ and the values of image intensity for each pixel (x, y) are independent, thus the conditional probability mentioned is

$$\prod_{(x,y)\in\Omega} P\left(U_{x,y} = u_{x,y} \mid U_{x,y} = u_{x,y}\right),$$

where $u_{x,y} = u(x, y)$ and $\tilde{u}_{x,y} = \tilde{u}(x, y)$. *Bayes' Rule* provides that

$$P\left(U_{x,y} = u_{x,y} \mid \tilde{U}_{x,y} = \tilde{u}_{x,y}\right) = \frac{P\left(\tilde{U}_{x,y} = \tilde{u}_{x,y} \mid U_{x,y} = u_{x,y}\right) P\left(U_{x,y} = u_{x,y}\right)}{P\left(\tilde{U}_{x,y} = \tilde{u}_{x,y}\right)},$$

where, at point (x, y), $P\left(U_{x,y} = u_{x,y} \mid \tilde{U}_{x,y} = \tilde{u}_{x,y}\right)$ is the conditional probability of the intensity of the noiseless image u on the condition of the intensity of the observed image u, $P\left(\tilde{U}_{x,y} = \tilde{u}_{x,y} \mid U_{x,y} = u_{x,y}\right)$ is the conditional probability of the intensity of the observed image u on the condition of the intensity of the noiseless image u, $P(U_{x,y} = u_{x,y})$ is the probability of intensity of the noiseless image and $P\left(\tilde{U}_{x,y} = \tilde{u}_{x,y}\right)$ is the probability of intensity of the observed image u. In order to maximize $P\left(U_{x,y} = u_{x,y} \mid \tilde{U}_{x,y} = \tilde{u}_{x,y}\right)$, we are going to find the noiseless image u which maximizes

$$\prod_{(x,y)\in\Omega} P\left(\tilde{U}_{x,y} = \tilde{u}_{x,y} \mid U_{x,y} = u_{x,y}\right) P\left(U_{x,y} = u_{x,y}\right). \tag{1}$$

For each (x, y), u is a Rayleigh random variable and its probability density is

$$P\left(\tilde{U}_{x,y} = \tilde{u}_{x,y}\right) = R_\sigma(\tilde{u}_{x,y}) = \frac{\tilde{u}_{x,y}}{\sigma^2} e^{-\frac{\left[\tilde{u}_{x,y}\right]^2}{2\sigma^2}}.$$

Assume the parameter σ of the conditional probability of u on the condition of the noiseless image u is a function of u, $\sigma = \sigma(u)$. Expression (1) becomes

$$\left(\prod_{(x,y)\in\Omega} \frac{\tilde{u}_{x,y}}{\left[\sigma\left(u_{x,y}\right)\right]^2} e^{-\frac{\left[\tilde{u}_{x,y}\right]^2}{2\left[\sigma\left(u_{x,y}\right)\right]^2}} \right) P\left(U_{x,y} = u_{x,y}\right). \tag{2}$$

In logarithm form, the problem of maximizing expression (2) is equivalent to the problem of minimizing

$$\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left(\frac{[\tilde{u}_{x,y}]^2}{2[\sigma(u_{x,y})]^2}+2\ln\sigma(u_{x,y})-\ln\tilde{u}_{x,y}\right)-\ln\left(\prod_{(x,y)\in\Omega}P(U_{x,y}=u_{x,y})\right).$$

We regard it as a discrete approximation of the functional

$$E(u)=-\ln P(u)+\iint_{\Omega}\left(\frac{\tilde{u}^2}{2[\sigma(u)]^2}+2\ln\sigma(u)-\ln\tilde{u}\right)dA,$$

where $P(u)$ is the probability that the random variable $U_{x,y}$ is equal to the intensity of the noiseless image u at pixel (x,y) for all $(x,y)\in\Omega$. For the model of a variational approach, Green [5] presents that $P(u)$ is given by

$$P(u)=e^{-\beta\iint_{\Omega}\sqrt{u_x^2+u_y^2}\,dA},$$

where β is a parameter. Hence, functional $E(u)$ becomes

$$E(u)=\beta\iint_{\Omega}\sqrt{u_x^2+u_y^2}\,dA+\iint_{\Omega}\left(\frac{\tilde{u}^2}{2[\sigma(u)]^2}+2\ln\sigma(u)-\ln\tilde{u}\right)dA.$$

The Euler-Lagrange equation for minimizing $E(u)$ is

$$\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)+\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right)+\frac{\sigma'(u)}{\beta[\sigma(u)]^3}\left(\tilde{u}^2-2[\sigma(u)]^2\right)=0.$$

Since the last term of this equation is a data-fidelity term [3], it vanishes when $u=u$:

$$\frac{\sigma'(\tilde{u})}{\beta[\sigma(\tilde{u})]^3}\left(\tilde{u}^2-2[\sigma(\tilde{u})]^2\right)=0.$$

For simplicity, the function $\sigma(u)=\frac{u}{\sqrt{2}}$ is chosen for satisfying this requirement. The functional $E(u)$ obtained is

$$E(u)=\beta\iint_{\Omega}\sqrt{u_x^2+u_y^2}\,dA+\iint_{\Omega}\left(\frac{\tilde{u}^2}{u^2}+2\ln u-\ln\sqrt{2}-\ln\tilde{u}\right)dA.$$

The Euler-Lagrange equation for minimizing $E(u)$ is

$$\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)+\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right)+\frac{2}{\beta u^3}(\tilde{u}^2-u^2)=0,$$

where $u=\tilde{u}$ on $\partial\Omega$.

D. Numerical Results

To verify the theoretical part, we use some images in our experiments. The correlation coefficients of the original images and the noisy images are compared with the correlation coefficients of the original images and the reconstructed images.

First, speckle noise with 0.02 variance is added to the original pattern image by MATLAB software version 7.2. Correlation coefficients of the original image and the noisy image is 0.9678 while correlation coefficients of the original image and reconstructed images with respective to the number of iterative loops are shown in Table 1. They are higher than 0.9678.

Table 1 Correlation coefficients of reconstructed pattern images

Iterative Loops	ROF Model	Model by Le et al.	Model by Aubert and Aujol	Proposed model
0	0.9678	0.9678	0.9678	0.9678
200	0.9882	0.9960	0.9963	0.9963
250	0.9889	0.9971	0.9974	0.9974
300	0.9893	0.9978	0.9980	0.9980
350	0.9895	0.9981	0.9982	0.9982

(a) Original Pattern image

(b) Speckle noisy pattern image.

(c) Image reconstructed by ROF model (200 loops)

(d) Image reconstructed by Le et al. model (200 loops)

(e) Image reconstructed by Aubert and Aujol model (200 loops)

(f) Image reconstructed by the proposed model (200 loops)

Fig 1 Original pattern image, Speckle noisy pattern image, and images reconstructed by a variety of methods for 200 loops

Furthermore, we use the Lenna image which is a well-known image in the field of image processing in our experiment. Speckle noise with 0.02 variance is added in the original image by MATLAB software. Similarly, correlation coefficients are compared and they are shown in Table 2. The correlation coefficient of the original image and noisy image is 0.9444 while the correlation coefficients of the original image and reconstructed images are all higher.

Table 2 Correlation coefficients of reconstructed Lenna images

Iterative Loops	ROF Model	Model by Le et al.	Model by Aubert and Aujol	Proposed model
0	0.9444	0.9444	0.9444	0.9444
80	0.9663	0.9725	0.9730	0.9730
120	0.9704	0.9798	0.9804	0.9804
160	0.9728	0.9843	0.9848	0.9848
200	0.9743	0.9868	0.9870	0.9871

(a) Original Lenna image

(b) Speckle noisy Lenna image.

(c) Image reconstructed by ROF model (100 loops)

(d) Image reconstructed by Le et al. model (100 loops)

(e) Image reconstructed by Aubert and Aujol model (100 loops)

(f) Image reconstructed by the proposed model (100 loops)

Fig 2 Original Lenna image, Speckle noisy Lenna image, and images reconstructed by a variety of methods for 100 loops

The output of the prototype software shows that after enhancing the ultrasound image, one obtains a smoother image as presented in figure 3.

(a) Original ultrasound image (Provided by Dr.Chumrus Sakulpaisarn)

(b) Ultrasound image reconstructed by the proposed model (100 loops)

Fig 3 Ultrasound image and the image reconstructed by the proposed method (100 loops)

The results show that the correlation coefficients of the original images and the reconstructed images are closer to 1 than those of the original and the noisy images. This establishes experimentally the validity of our model.

III. Conclusions

A mathematical model for noise removal in ultrasound images has been developed, which helps for enhancing ultrasound images with speckle noise. Numerical tests have shown that such images can be denoised with good success by employing this model. The application of this theory to computer software will support clinicians in diagnosis of digital ultrasound images.

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