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Symmetry analysis of the nonhomogeneous inviscid Burgers equation with delay

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ABSTRACT

Many mathematical models in science are described by delay differential equations. Recent developments of the theory of delay differential equations allow one to derive a method for studying this class of equations by the group analysis method. So far there have been few investigations of delay differential equations by group analysis method. The present article studies the delay partial differential equation

$$\frac{\partial u}{\partial t}(x,t)+u(x,t)\frac{\partial u}{\partial x}(x,t)=G(u(x,t-\tau),u(x,t)).$$

The complete group classification of this delay equation with the functional $G=\mathcal{G}(u(x,t)-u(x,t-\tau))+\mathcal{H}(u)$ is given in this article. The classification is considered with respect to the functions $\mathcal{G}$ and $\mathcal{H}$.

1. Introduction

Group analysis is one of the methods for solving differential equations. It has been applied to many mathematical models, and is used not only for solving differential equations, but also for developing new models. A survey of applications of group analysis in science can be found in [1]. In modern science, one encounters many mathematical models described by delay differential equations [2–4]. As the group analysis method was mostly developed for its applications to differential equations, it is necessary to generalize this method for applications to delay differential equations. Such a generalization was derived in [5–7]. There are a few applications of this generalization to delay differential equations [8,5,6,9–11]. For partial differential equations, the group analysis method finds symmetries of the equation. The existence of a symmetry provides the construction of an invariant solution which is able to transform the original system of equations to a system of equations with fewer numbers of independent variables. However, for delay differential equation, such type of theorem has not been developed yet. The present article provides a new application of the developing theory. It shows that the presence of a symmetry of a delay differential equation allows finding an invariant solution and reducing a number of independent variables of the equation, which is similar to the theory for partial differential equations. It is also worth to note that the method in [5–7] is similar to the concept of generalization of the group analysis method to numerical schemes [12,13].

1.1. The proposed equation

The partial differential equation

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