จงหาผลเฉลยทั่วไปของ

$$\underbrace{\left(3x^2+4xy\right)}\,dx+\underbrace{\left(2x^2+2y\right)}\,dy=0$$

$M = 3x^2+4xy$

$N = 2x^2+2y$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(\cancel{3x^2}+4xy\right)$$

$$= 4x\frac{\partial y}{\partial y}$$

$$= 4x$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(2x^2+\cancel{2y}\right)$$

$$= 4x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

ดังนั้น สมการนี้เป็นสมการแบบแม่นตรง

$$f(x,y) = C$$

$$\frac{\partial f}{\partial x} = M, \qquad \frac{\partial f}{\partial y} = N$$

$$f = \int M\,dx$$

$$= \int (3x^2 + 4xy)\,dx$$

$$= \int 3x^2\,dx + \int 4xy\,dx$$

$$= 3\int x^2\,dx + 4y\int x\,dx$$

$$= 3\frac{x^3}{3} + 4y\frac{x^2}{2} + g(y)$$

$$f = x^3 + 2x^2y + \boxed{g(y)}$$

$$f = x^3 + 2x^2y + \boxed{g(y)}$$

$$\frac{\partial f}{\partial y} = N = 2x^2 + 2y$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^3 + 2x^2y + g(y)\right)$$

$$= \cancel{\frac{\partial x^2}{\partial y}} + \frac{\partial (2x^2y)}{\partial y} + \frac{dg(y)}{dy}$$

$$= 0 + 2x^2\frac{\partial y}{\partial y} + g'(y)$$

$$= 2x^2 + g'(y) = N$$

$$\cancel{2x^2} + g'(y) = \cancel{2x^2} + 2y$$

$$g'(y) = 2y$$

$$g'(y) = 2y$$

$$\boxed{g(y)} = \int 2y\,dy = \frac{2y^2}{2} \cancel{\boxed{+C}}$$

$$= y^2$$

$$\boxed{f(x,y) = x^3 + 2x^2y + y^2} = C$$

$$f(x,y) = c$$

$$\frac{\partial f}{\partial x} = M, \qquad \boxed{\frac{\partial f}{\partial y} = N}$$

$$f = \int N\,dy$$

$$= \int (2x^2 + 2y)\,dy$$

$$= \int 2x^2\,dy + \int 2y\,dy$$

$$= 2x^2 \int 1\,dy + 2\int y\,dy$$

$$= 2x^2 y + 2\frac{y^2}{2} + h(x)$$

$$f = 2x^2 y + y^2 + h(x)$$

$$f = \underline{2x^2y + y^2} + \boxed{h(x)}$$

$$\frac{\partial f}{\partial x} = M$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(2x^2y + y^2 + h(x)\right)$$

$$= 2 \cdot 2x \cdot y + 0 + h'(x)$$

$$= \cancel{4xy} + h'(x) = M = 3x^2 + \cancel{4xy}$$

$$h'(x) = 3x^2$$

$$h(x) = \int 3x^2\,dx$$

$$= x^3$$

$$f(x,y) = 2x^2y + y^2 + x^3 = C$$

จงหาผลเฉลยทั่วไปของ

$$\underbrace{(ye^{xy} + \sin y)}\,\underline{dx} + \underbrace{(xe^{xy} + x\cos y)}\,dy = 0$$

$M = ye^{xy} + \sin y$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(ye^{xy} + \sin y)$

$= \frac{\partial}{\partial y}(ye^{xy}) + \frac{\partial}{\partial y}(\sin y)$

$= y\frac{\partial e^{xy}}{\partial y} + e^{xy}\frac{\partial y}{\partial y} + \cos y$

$= xye^{xy} + e^{xy} + \cos y$

$N = xe^{xy} + x\cos y$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(xe^{xy} + x\cos y)$

$= \frac{\partial}{\partial x}(xe^{xy}) + \frac{\partial}{\partial x}(x\cos y)$

$= x\frac{\partial e^{xy}}{\partial x} + e^{xy}\cancel{\frac{\partial x}{\partial x}} + \cos y\cancel{\frac{\partial x}{\partial x}}$

$= xye^{xy} + e^{xy} + \cos y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

สมการดังกล่าวเป็นสมการแบบแม่นตรง

$$f = \int M \, dx = \int (ye^{xy} + \sin y) \, dx$$

$$= \int ye^{xy} \, dx + \int \sin y \, dx$$

$$= y \int e^{xy} \, dx + (\sin y) \int 1 \, dx$$

$$= \cancel{y} \frac{e^{xy}}{\cancel{y}} + (\sin y) x + g(y)$$

$$f = e^{xy} + x \sin y + \boxed{g(y)}$$

$$f = e^{xy} + x\sin y + \boxed{g(y)}$$

$$\frac{\partial f}{\partial y} = N$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(e^{xy} + x\sin y + g(y)\right)$$

$$= \frac{\partial e^{xy}}{\partial y} + \frac{\partial}{\partial y}(x\sin y) + \frac{\partial}{\partial y} g(y)$$

$$= x e^{xy} + x\frac{\partial \sin y}{\partial y} + g'(y)$$

$$= \cancel{xe^{xy}} + \cancel{x\cos y} + g'(y) = N$$

$$= \cancel{xe^{xy}} + \cancel{x\cos y}$$

$$g'(y) = 0$$

$$g(y) = 0$$

$$f = e^{xy} + x\sin y + \boxed{g(y)}$$

$$f = e^{xy} + x\sin y = C$$

จงหาผลเฉลยของปัญหาค่าตั้งต้นต่อไปนี้

$$y' = \frac{-2xy}{1+x^2}, \quad y(2) = -5$$

$$\frac{dy}{dx} = \frac{-2xy}{1+x^2}$$

$$(1+x^2)\,dy = -2xy\,dx$$

$$2xy\,dx + (1+x^2)\,dy = 0$$

$$M = 2xy \qquad N = 1+x^2$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy) = 2x\frac{\partial y}{\partial y} = 2x$$

$$\frac{\partial N}{\partial x} = \frac{\partial(1+x^2)}{\partial x} = \frac{\partial 1}{\partial x} + \frac{\partial x^2}{\partial x} = 0 + 2x = 2x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

สมการดังกล่าวเป็นสมการแบบแม่นตรง

$$f = \int M \, dx$$

$$= \int (2xy) \, dx = 2y \int x \, dx$$

$$= \cancel{2} y \frac{x^2}{\cancel{2}} + g(y) = x^2 y + g(y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^2 y + g(y) \right) = x^2 + g'(y) = N = 1 + x^2$$

$$g'(y) = 1 + \cancel{x^2} - \cancel{x^2} = 1$$

$$g(y) = y$$

$$f(x,y) = x^2y + y = C$$

จาก $y(2) = -5$ $(x=2,\ y=-5)$

$$f(2,-5) = 2^2(-5) + (-5) = C$$

$$= 4\cdot(-5) - 5 = C$$

$$-20 - 5 = C$$

$$C = -25$$

$x^2y + y = -25$ เป็นผลเฉลยของปัญหาค่าเริ่มต้น

$$x^2y + y + 25 = 0$$

จงหาผลเฉลยของปัญหาค่าตั้งต้นต่อไปนี้

$$y' = \frac{-y^2}{2xy+1}, \boxed{y(1) = -2}$$

$$\frac{dy}{dx} = \frac{-y^2}{2xy+1}$$

$$(2xy+1)dy = -y^2 dx$$

$$y^2 dx + (2xy+1)dy = 0$$

$M = y^2$ $\quad$ $N = 2xy+1$

$$\frac{\partial M}{\partial y} = \frac{\partial y^2}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = \partial(2xy+1)$$

$$= 2y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

สมการดังกล่าวเป็นสมการแบบแม่นตรง

$$f = \int M\,dx$$

$$= \int y^2\,dx = y^2 \int 1\,dx = xy^2 + g(y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(xy^2 + g(y)) = \cancel{2xy} + g'(y) = N$$

$$= \cancel{2xy} + 1$$

$$g'(y) = 1$$

$$g(y) = \int 1\,dy = y$$

$$f = xy^2 + y = C$$

$$y(1) = -2 \quad (x=1, y=-2)$$

$$f(1,-2) = 1\cdot(-2)^2 + (-2) = C$$

$$4 - 2 = C$$

$$C = 2$$

ผลเฉลยของปัญหาค่าตั้งต้น

$$f(x,y) = xy^2 + y = 2$$

1. Simple equation

$$y' = f(x)$$

$$y = \int f(x)\,dx$$

2. Separable equation

$$\int h(y)\,dy = \int g(x)\,dx$$

3. Hogeneous equation

$$y' = f\left(\frac{y}{x}\right)$$

$9\,u^2$ $$v = \frac{y}{x} \Rightarrow y = xv \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} = f(v)$$

4. Linear equation

$$y' + p\,y = q$$

$p \qquad q$

$\int p\,dx$

$$y = e^{-\int p\,dx}\left[\int q\,e^{\int p\,dx}\,dx + C\right]$$

5. Bernoulli equation

$$y' + p \cdot y = q \cdot y^{n}$$

$$y^{-n} y' + p \cdot y^{1-n} = q$$

$$v = y^{1-n}$$

6. Exact equation สมการแบบแม่นตรง

$$M\,dx + N\,dy = 0$$

$M =$ $\qquad$ $N =$ - - -

$$\frac{\partial M}{\partial y} = \text{- - -} \qquad \frac{\partial N}{\partial x} = \text{- - .}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ เป็นสมการแบบแม่นตรง

$$f = \int M\,dx + g(y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\int M\,dx + g(y)\right] = N$$

$$f(x,y) = C$$