

$P(\rho, \phi, \theta)$  เป็นจุดใดๆ ในพิกัดทรงกลม

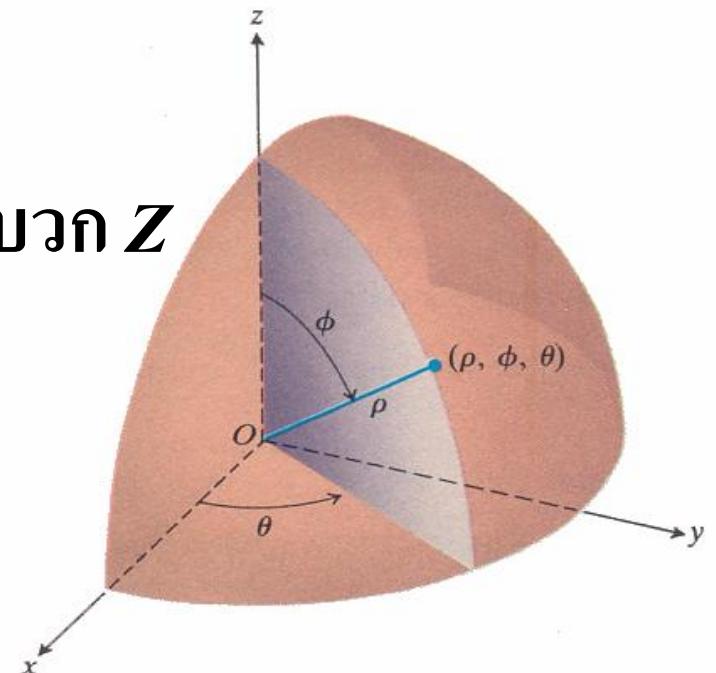
$\rho$  เป็นระยะจากจุดกำเนิดถึงจุด  $P$

$\phi$  เป็นมุมระหว่างเวกเตอร์  $\overrightarrow{OP}$  ถึงแกนบวก  $Z$

$$0 \leq \phi \leq \pi$$

$\theta$  เป็นมุมในพิกัดทรงกรวย

$$0 \leq \theta \leq 2\pi$$



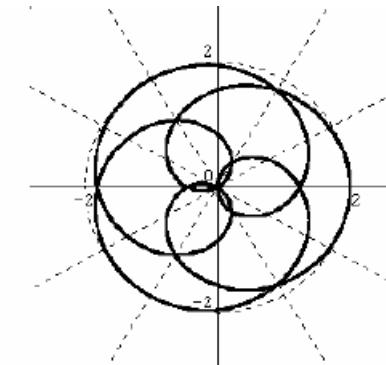
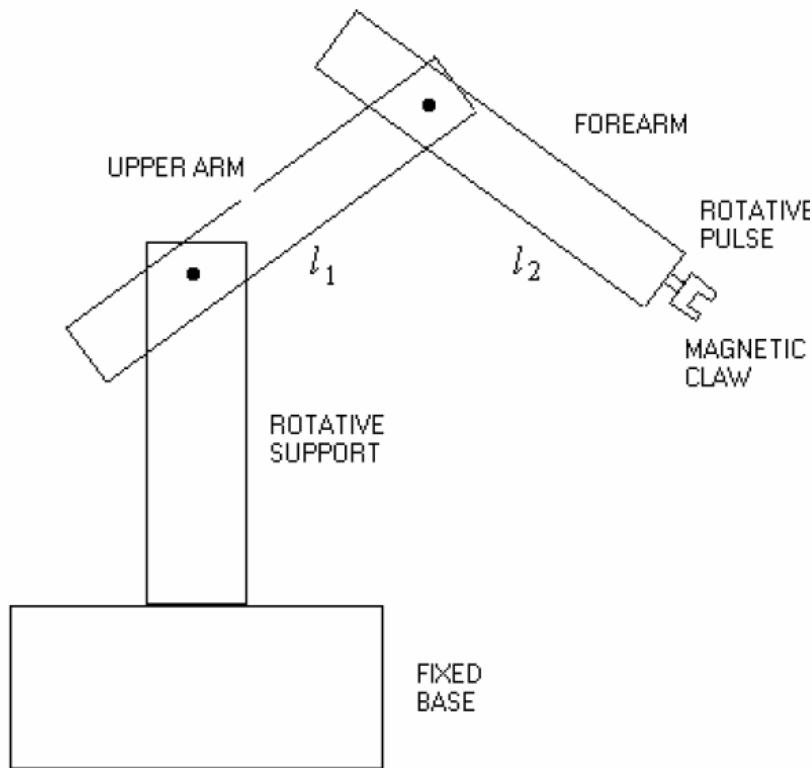


FIGURE. 3

PATTERN OF MOVEMENT OF THE CLAW WITH  $l_1=l_2=1$ ,  $w_1=2 \text{ rad/s}$  AND  $w_2=3 \text{ rad/s}$ .

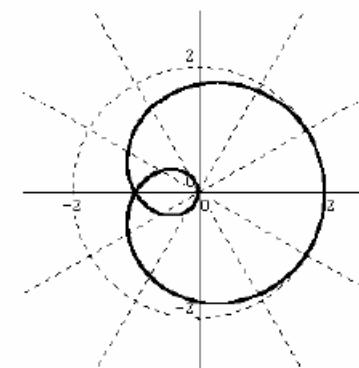
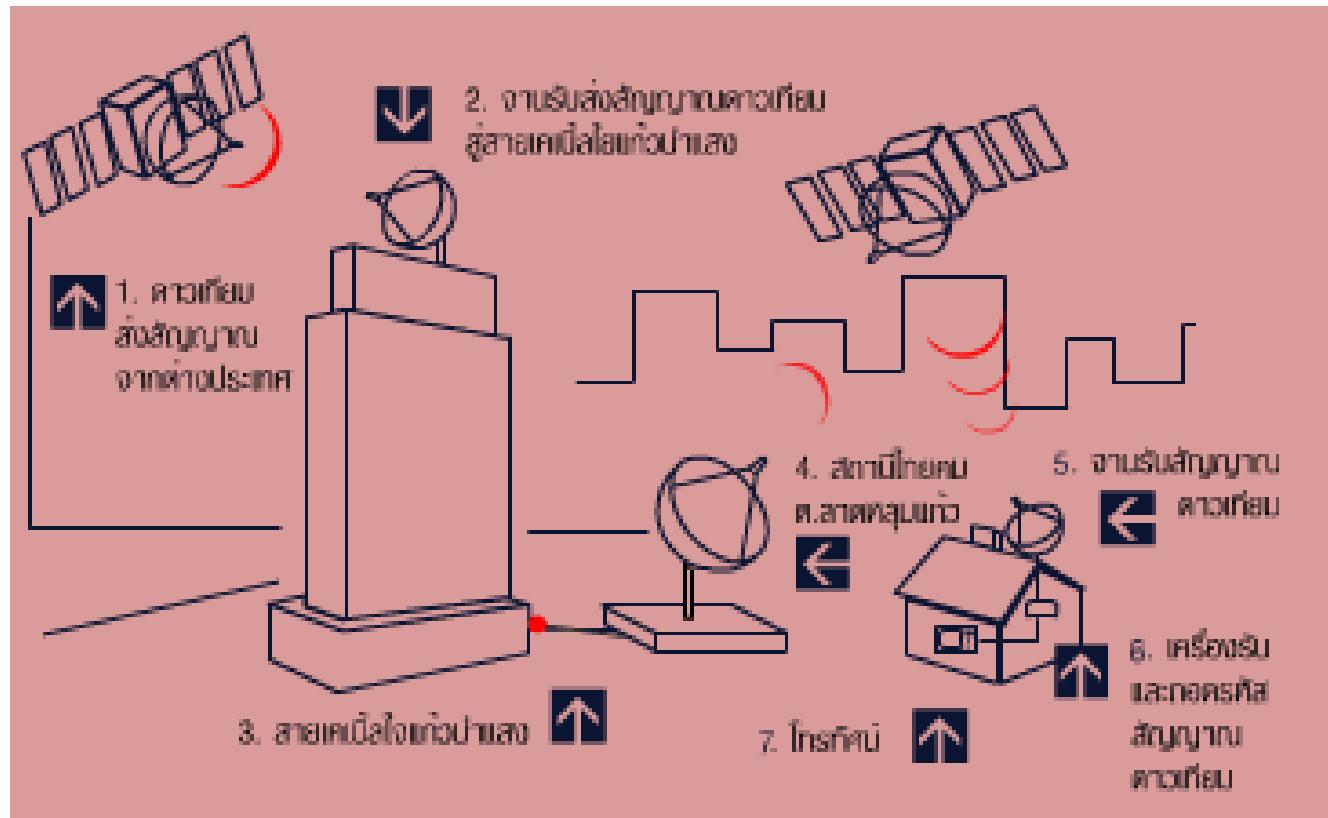
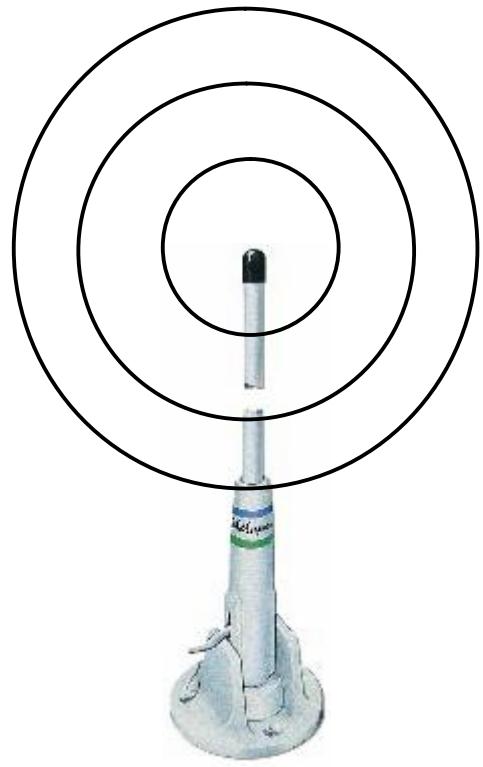


FIGURE. 4

PATTERN OF MOVEMENT OF THE CLAW WITH  $l_1=l_2=1$ ,  $w_1=w_2=1 \text{ rad/s}$ .

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\omega_1 * t) & \cos((\omega_1 + \omega_2) * t) \\ \sin(\omega_1 * t) & \sin((\omega_1 + \omega_2) * t) \end{bmatrix} * \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$$



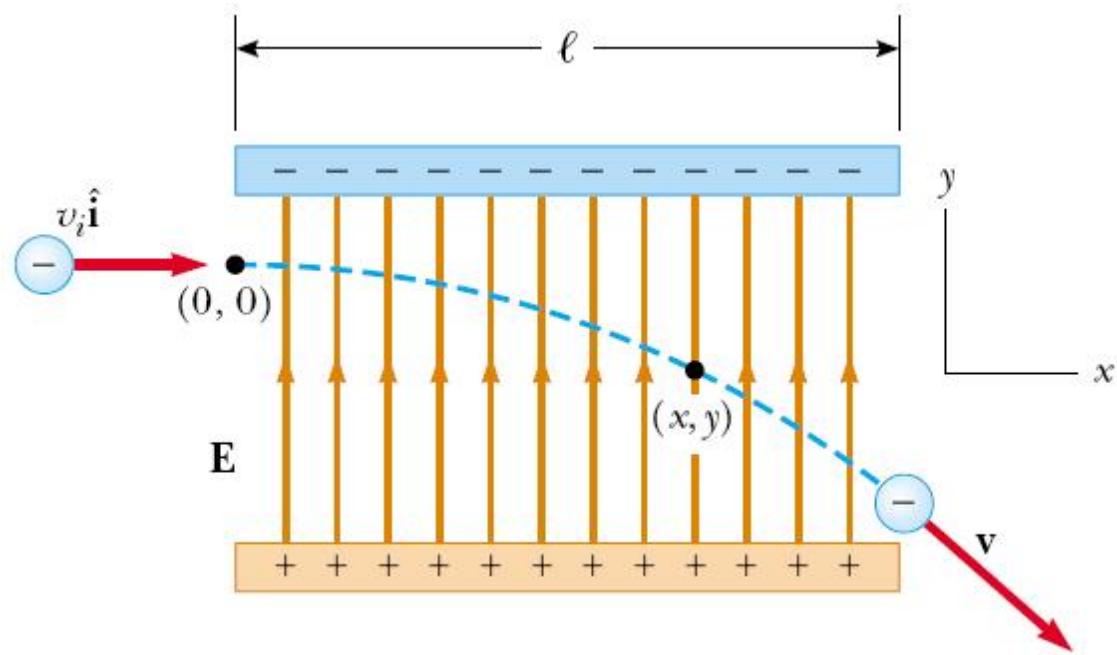
## **BASEBALL MACHINES**

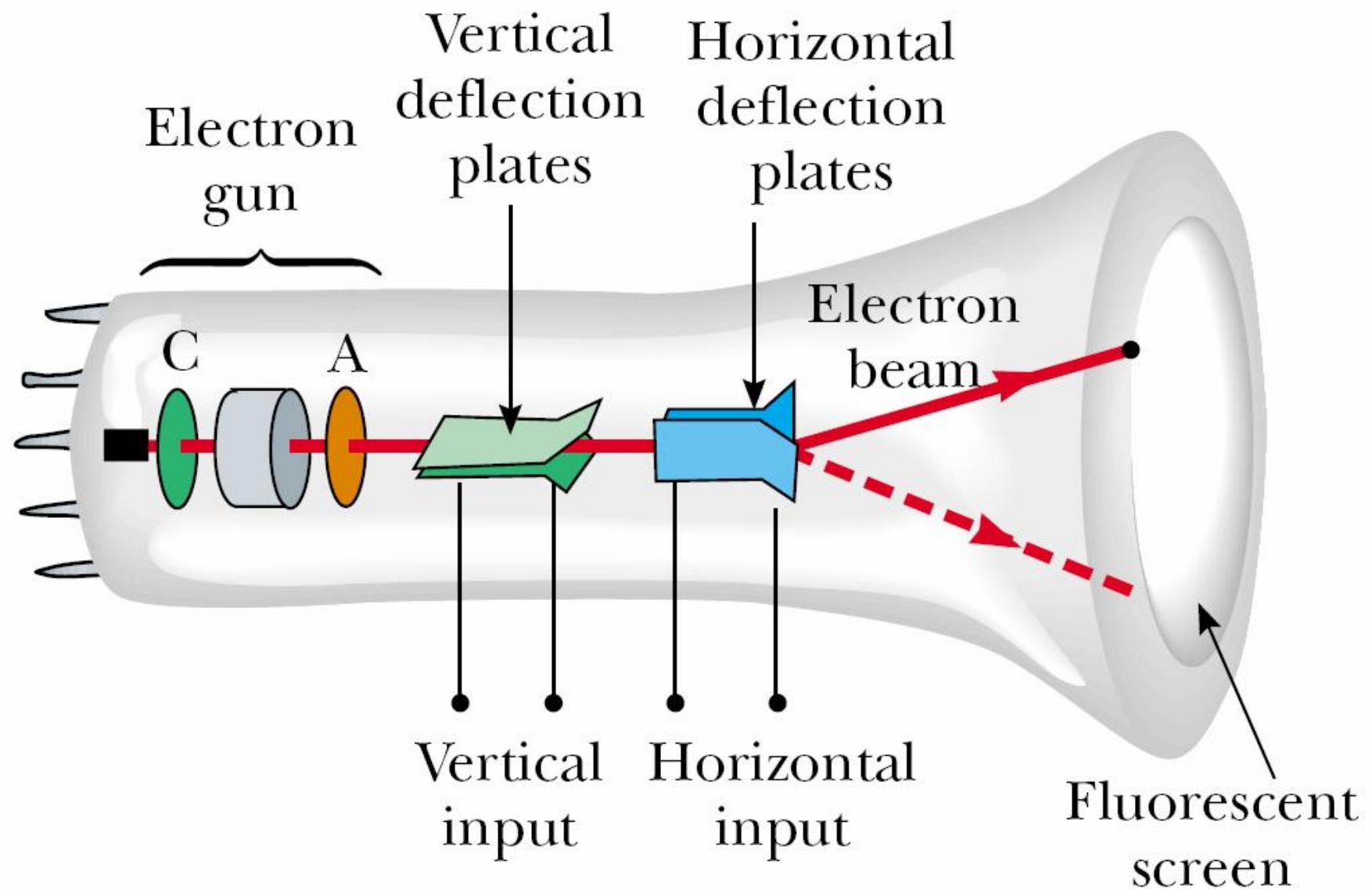
## VOLLEYBALL MACHINES

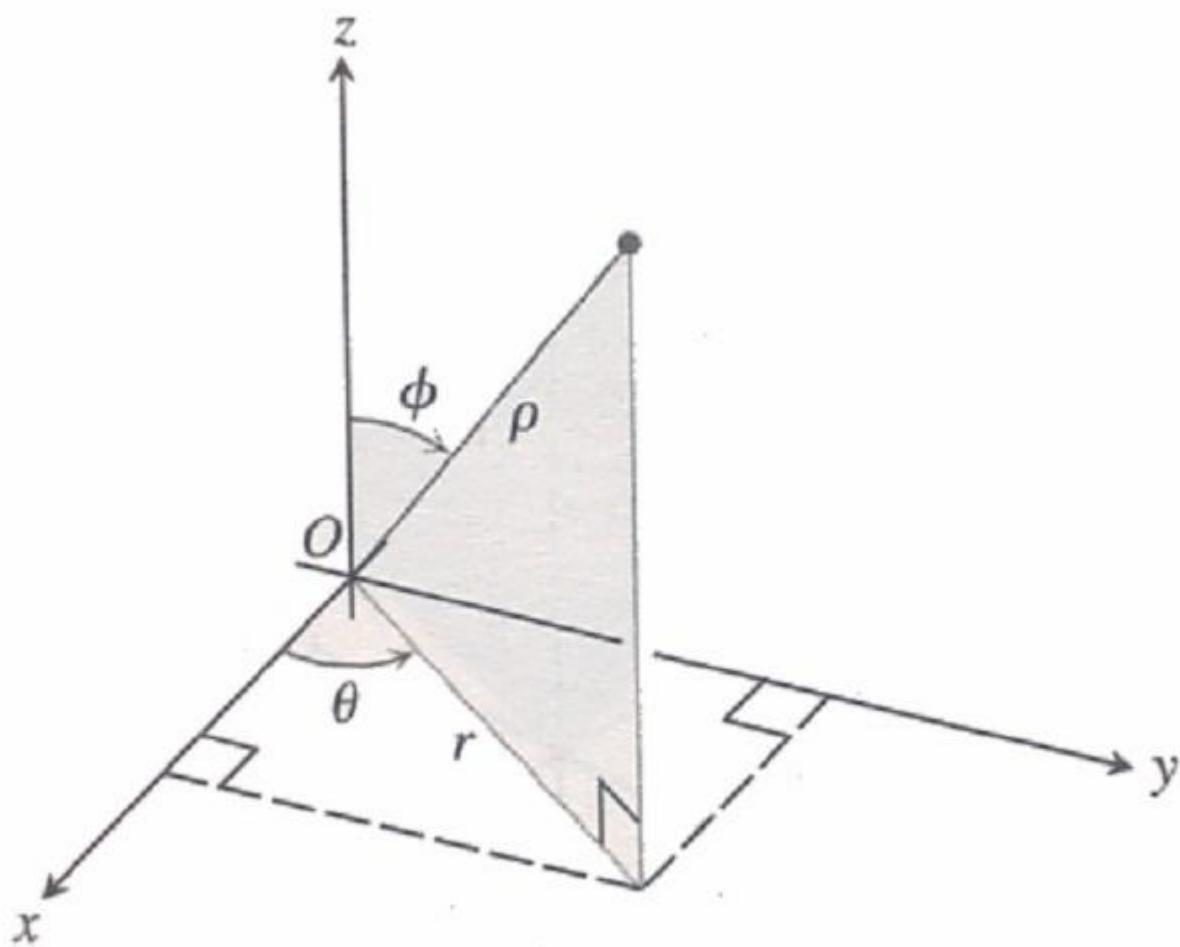
## SOCER MACHINES

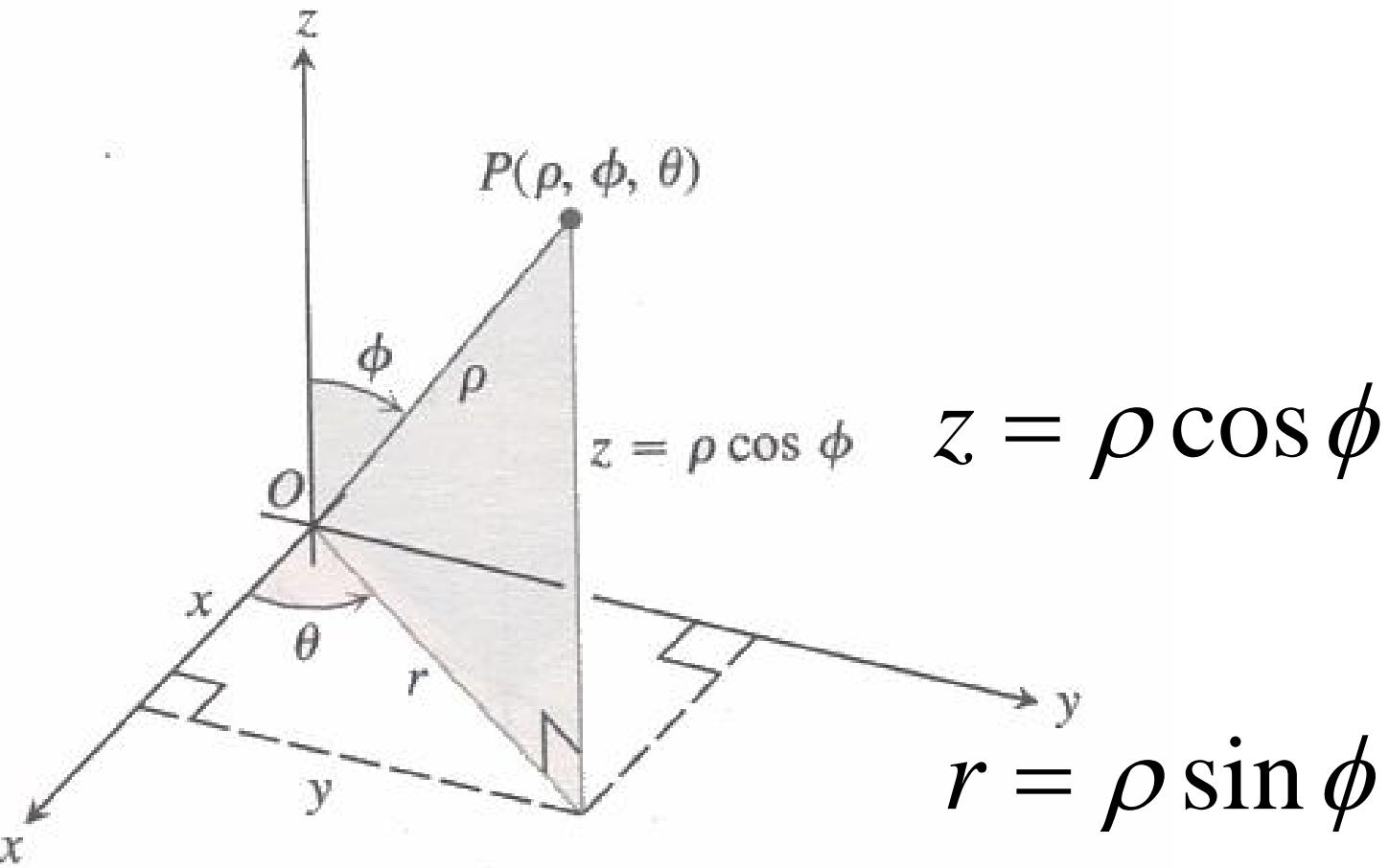
## TENNIS MACHINES







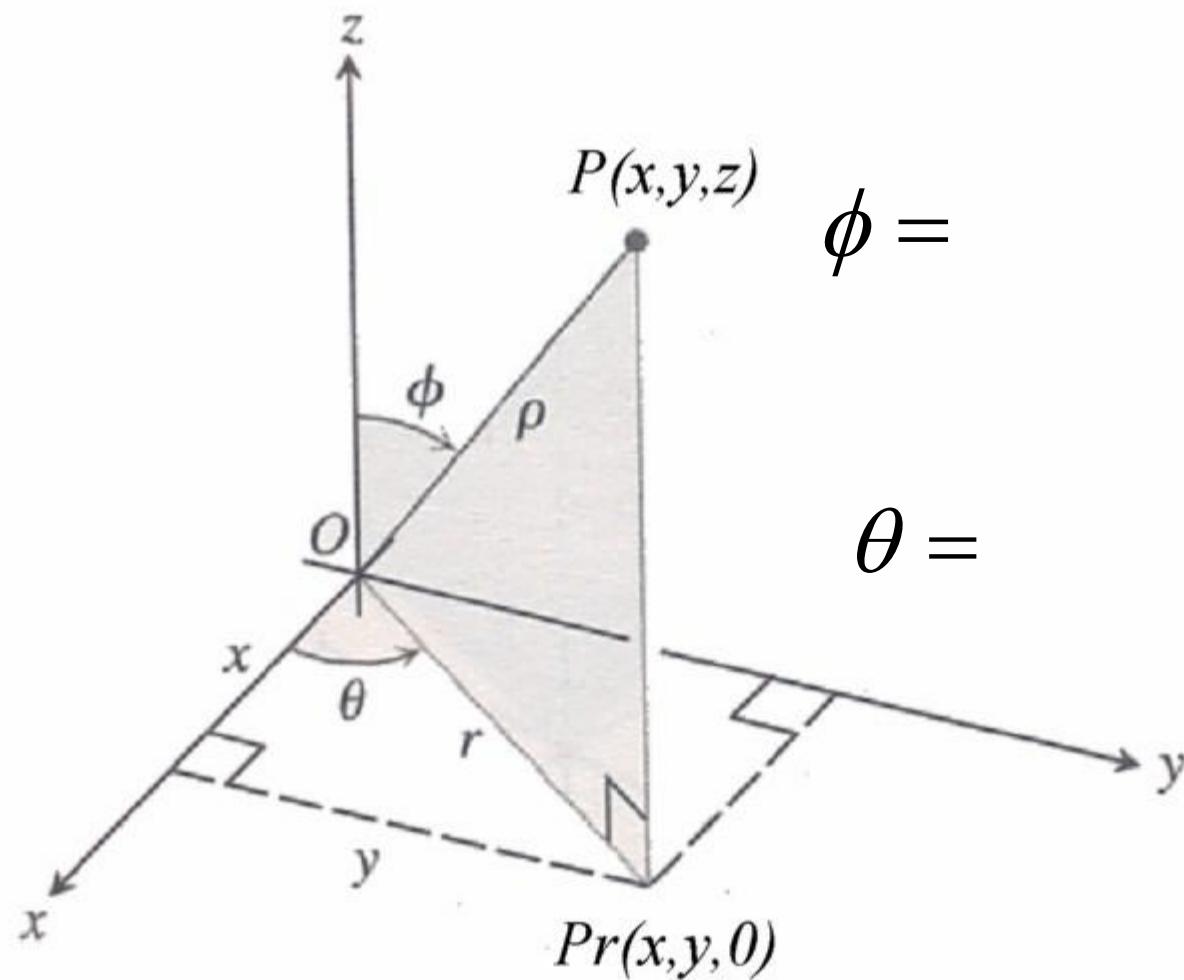




$$x = r \cos \theta = \rho \sin \phi \cos \theta$$

$$y = r \sin \theta = \rho \sin \phi \sin \theta$$

$$\rho =$$



$$\theta =$$

$$\tan \theta = \frac{y}{x} \qquad \qquad \theta = \tan^{-1} \frac{y}{x}$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$\frac{z}{\sqrt{x^2+y^2+z^2}}=\cos\phi$$

$$\phi = \cos^{-1} \left( \frac{z}{\sqrt{x^2+y^2+z^2}} \right)$$

จุด  $(x, y, z) = (3, \sqrt{3}, 2)$  ในพิกัด直角坐标 จุด ได  
ในพิกัดทรงกลม  $(r, \phi, \theta)$

# จงหาสมการในพิกัดทรงกลม ของทรงกลม

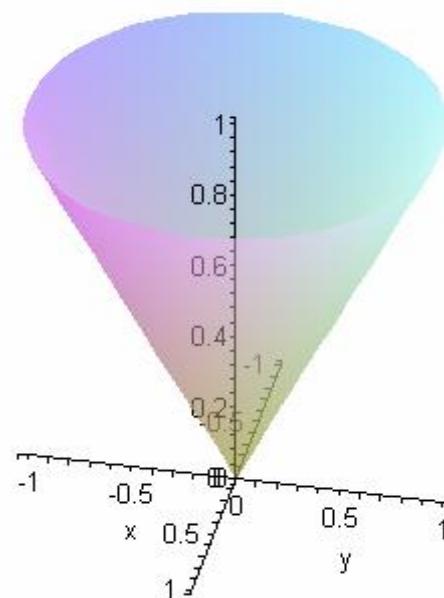
$$x^2 + y^2 + z^2 = 1$$

# จงหาสมการในพิภัติทรงกลม ของทรงกลม

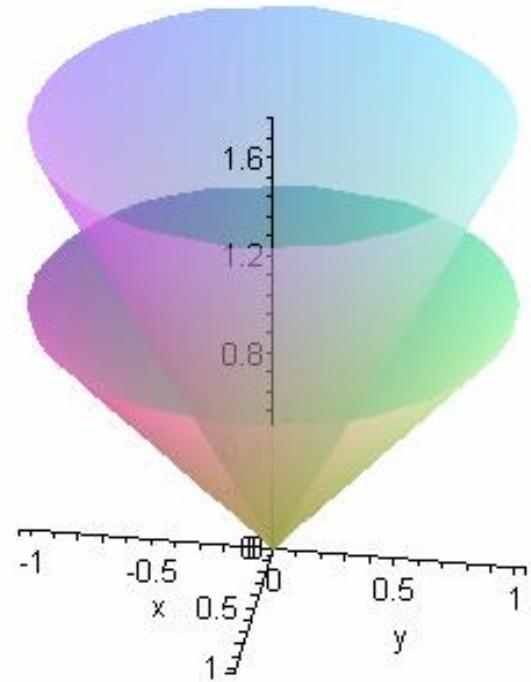
$$x^2 + y^2 + (z - 1)^2 = 1$$

# จงหาสมการในพิกัดทรงกลม ของกรวยกลม

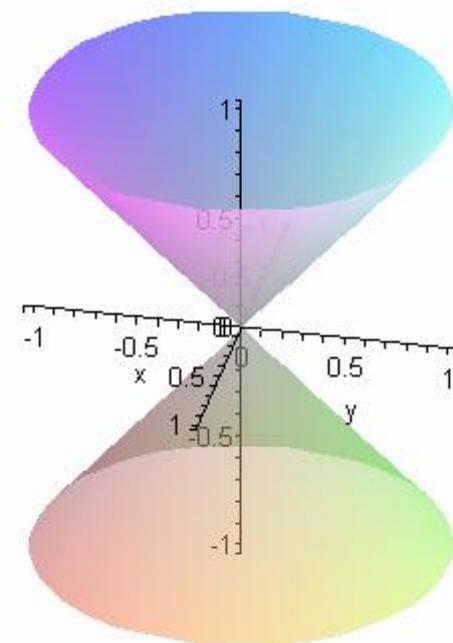
$$z = \sqrt{x^2 + y^2}$$



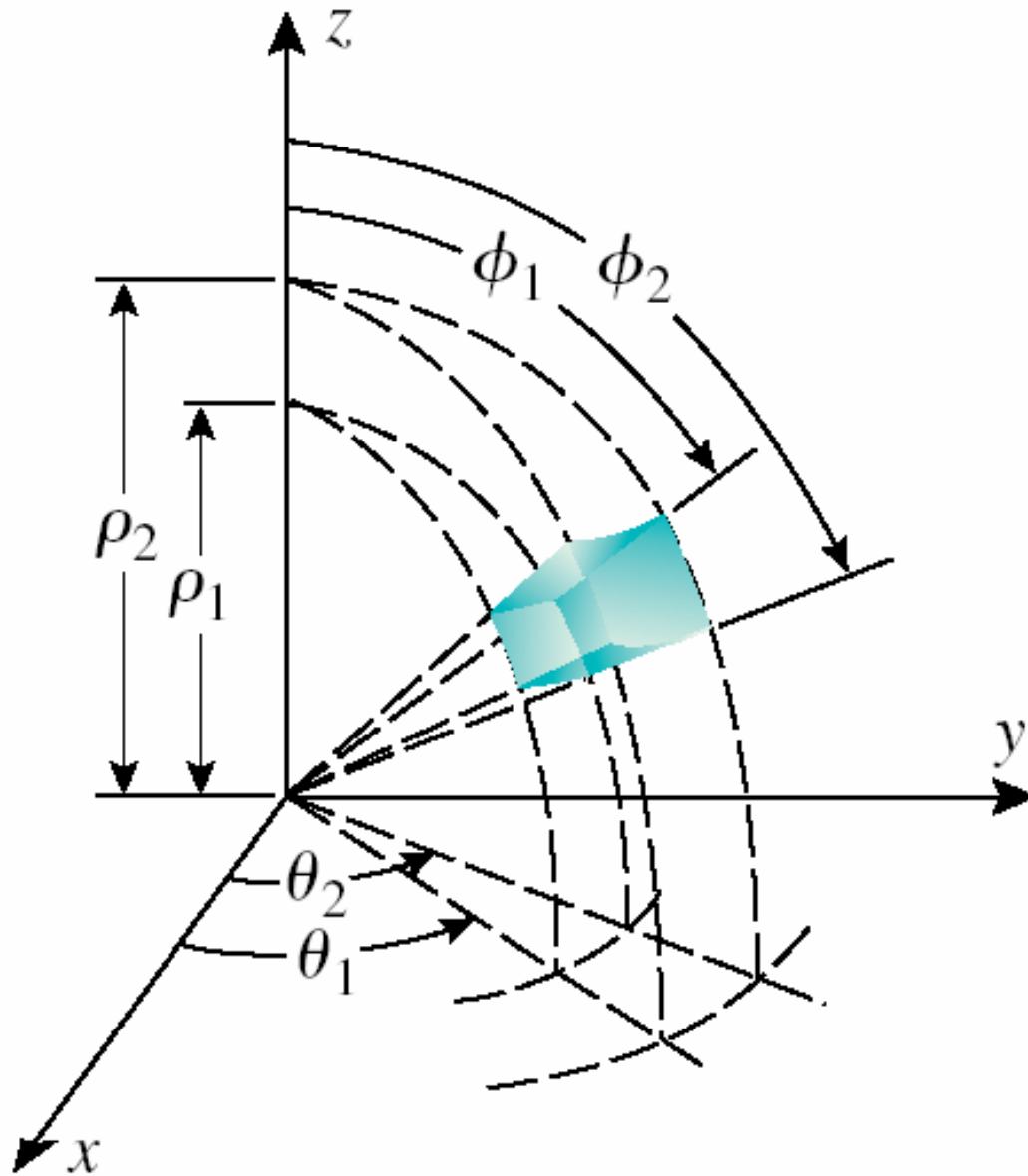
$$z = \sqrt{3(x^2 + y^2)}$$

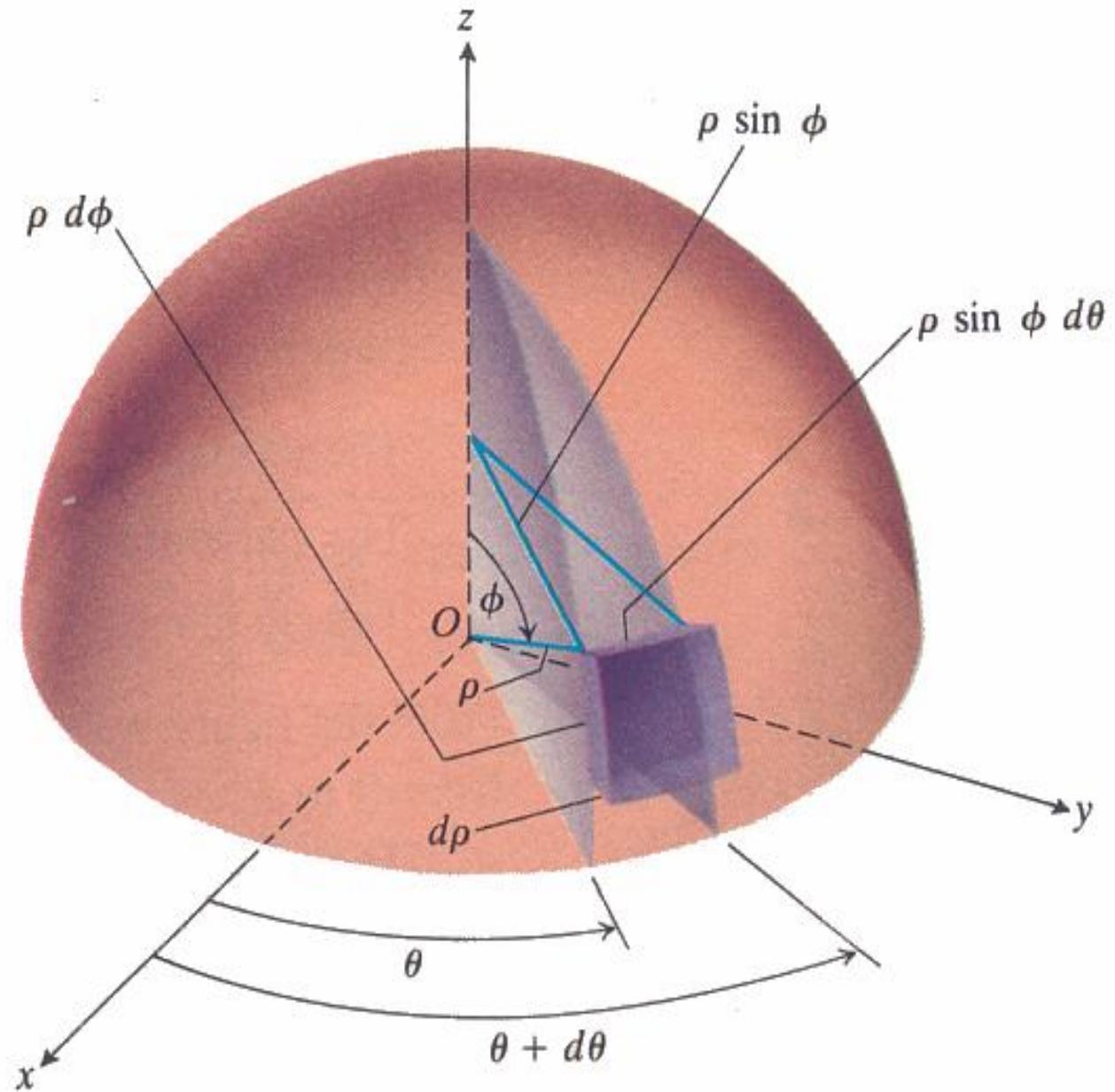


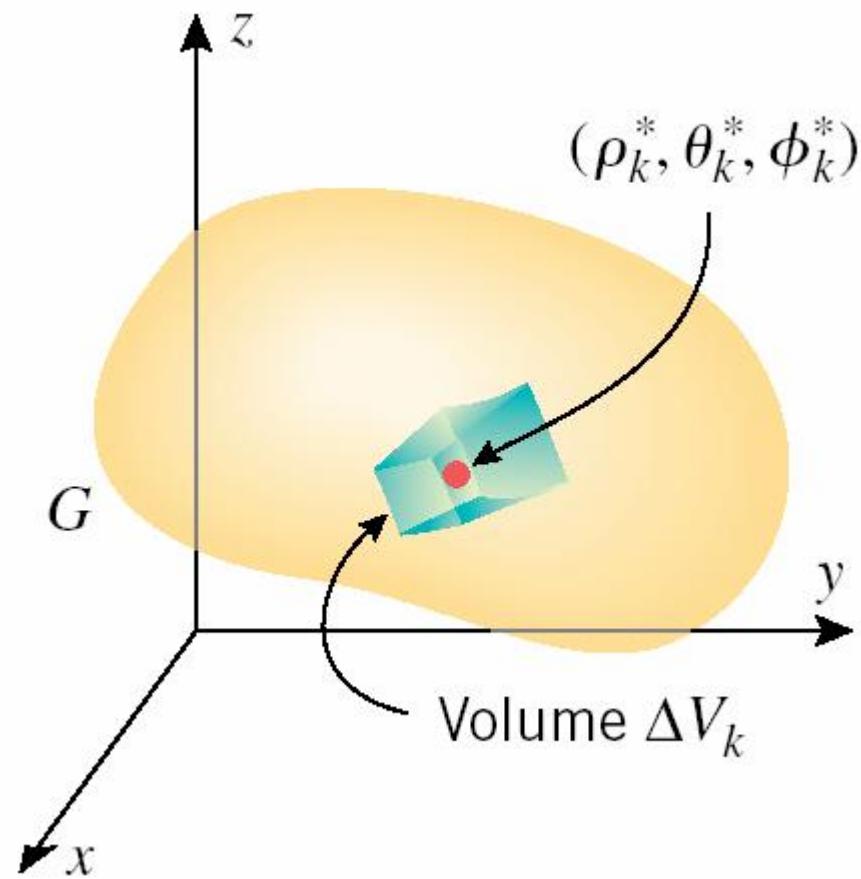
$$z^2 = x^2 + y^2$$



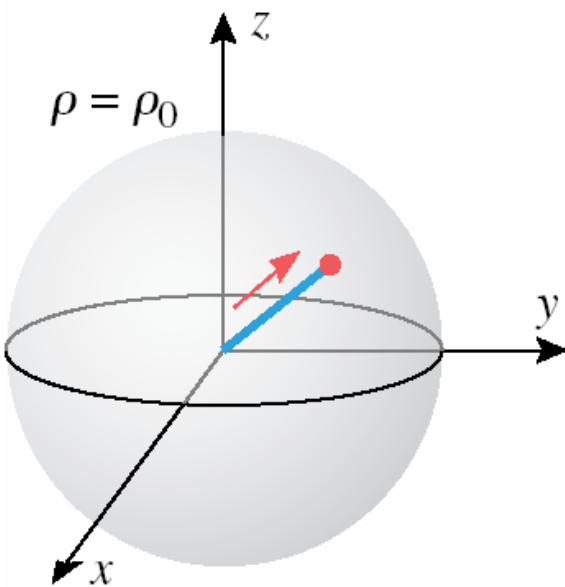
# อินทิกรัลในระบบพิกัดทรงกลม



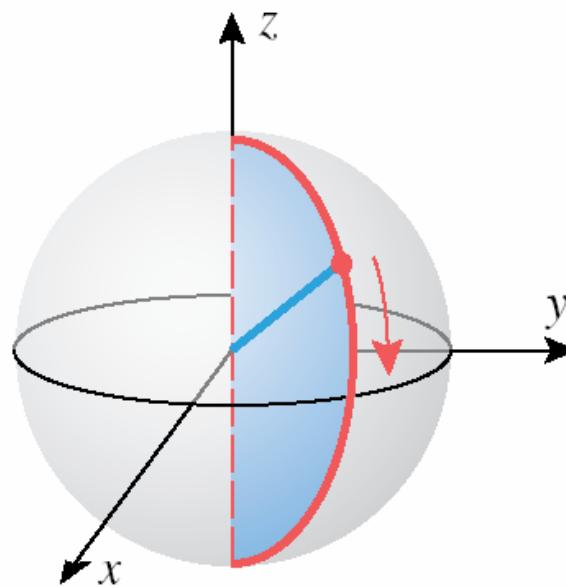




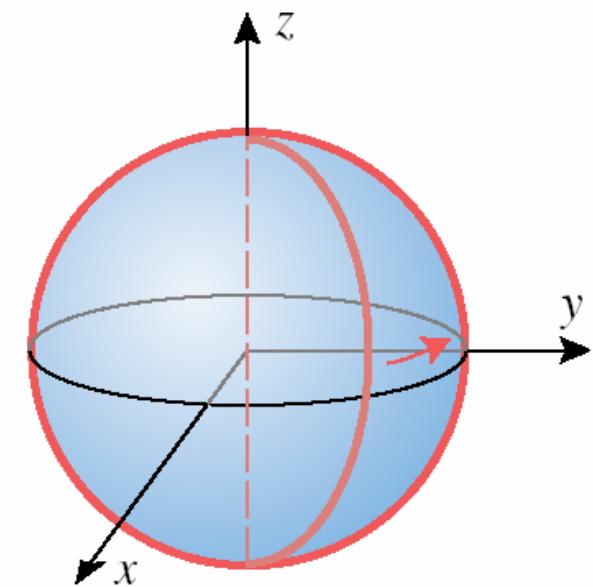
$$\iiint_G f(\rho, \theta, \phi) dV = \int_0^{2\pi} \int_0^{\pi} \int_0^{\rho_0} f(\rho, \theta, \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$



$\rho$  varies from 0 to  $\rho_0$   
with  $\theta$  and  $\phi$  fixed.



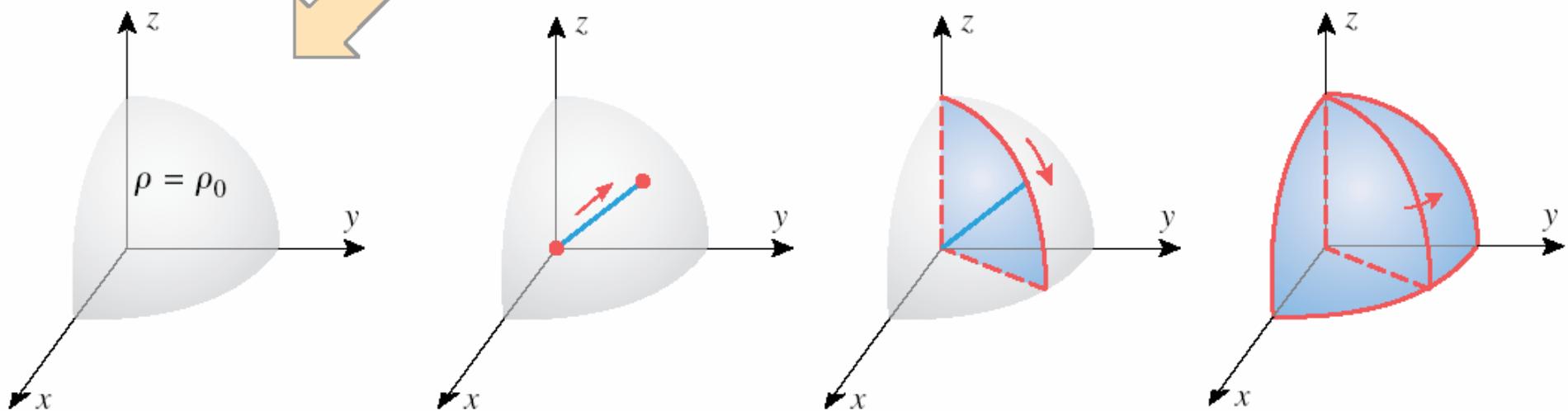
$\phi$  varies from 0 to  $\pi$   
with  $\theta$  fixed.



$\theta$  varies from 0 to  $2\pi$ .

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\rho_0} f(\rho, \theta, \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

This is the portion of the sphere of radius  $\rho_0$  that lies in the first octant.



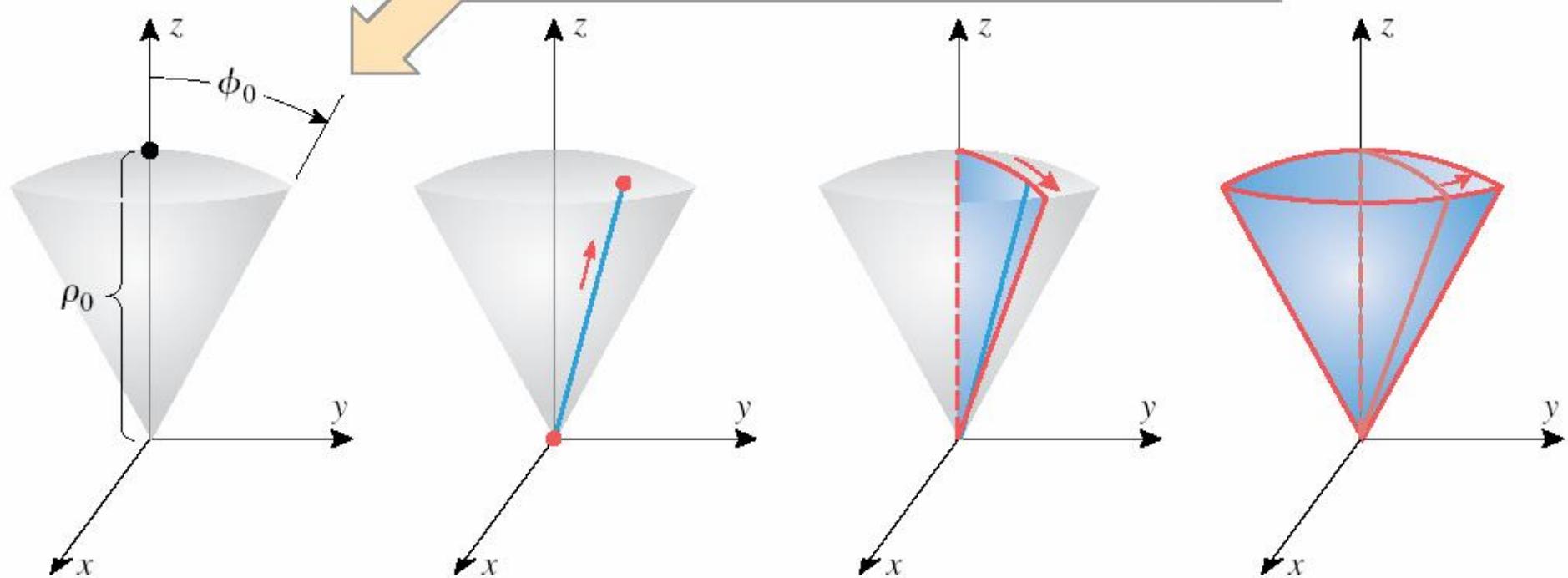
$\rho$  varies from 0 to  $\rho_0$  with  $\theta$  and  $\phi$  held fixed.

$\phi$  varies from 0 to  $\pi/2$  with  $\theta$  held fixed.

$\theta$  varies from 0 to  $\pi/2$ .

$$\int_0^{2\pi} \int_0^{\phi_0} \int_0^{\rho_0} f(\rho, \theta, \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

This ice-cream-cone-shaped solid is cut from the sphere of radius  $\rho_0$  by the cone  $\phi = \phi_0$ .



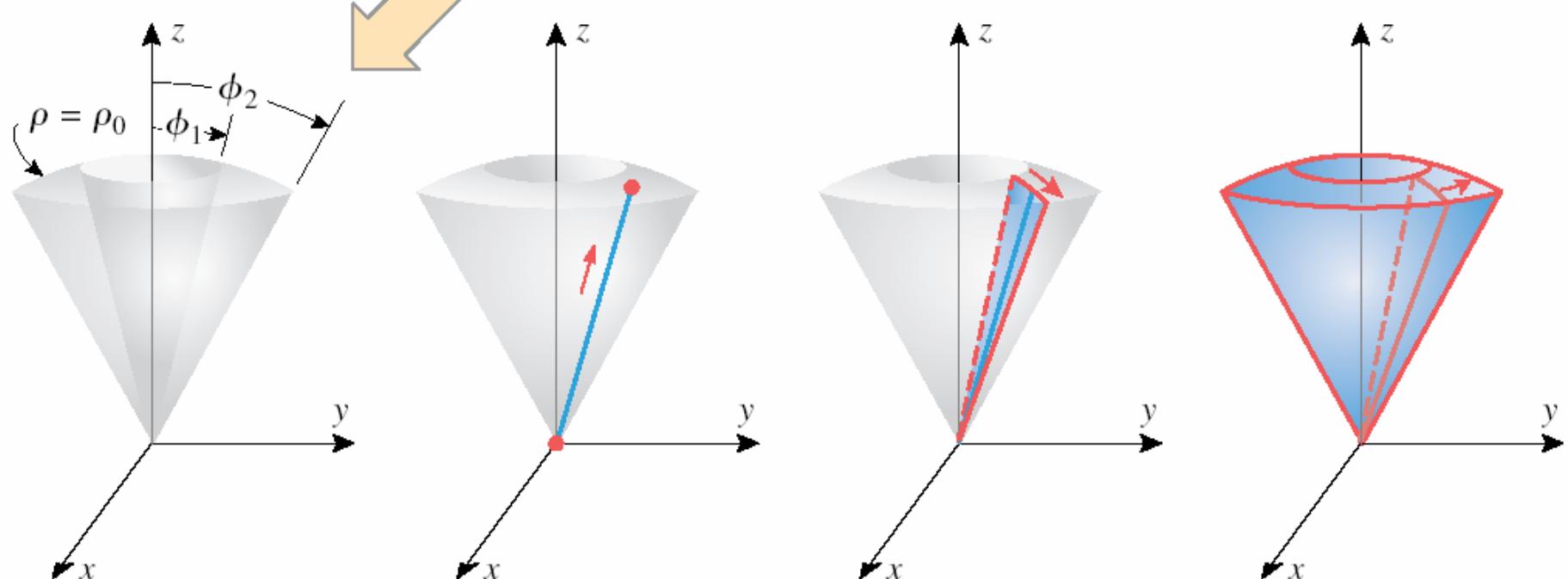
$\rho$  varies from 0 to  $\rho_0$  with  $\theta$  and  $\phi$  held fixed.

$\phi$  varies from 0 to  $\phi_0$  with  $\theta$  held fixed.

$\theta$  varies from 0 to  $2\pi$ .

$$\int_0^{2\pi} \int_{\phi_1}^{\phi_2} \int_0^{\rho_0} f(\rho, \theta, \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

This solid is cut from the sphere of radius  $\rho_0$  by two cones,  $\phi = \phi_1$  and  $\phi = \phi_2$ .



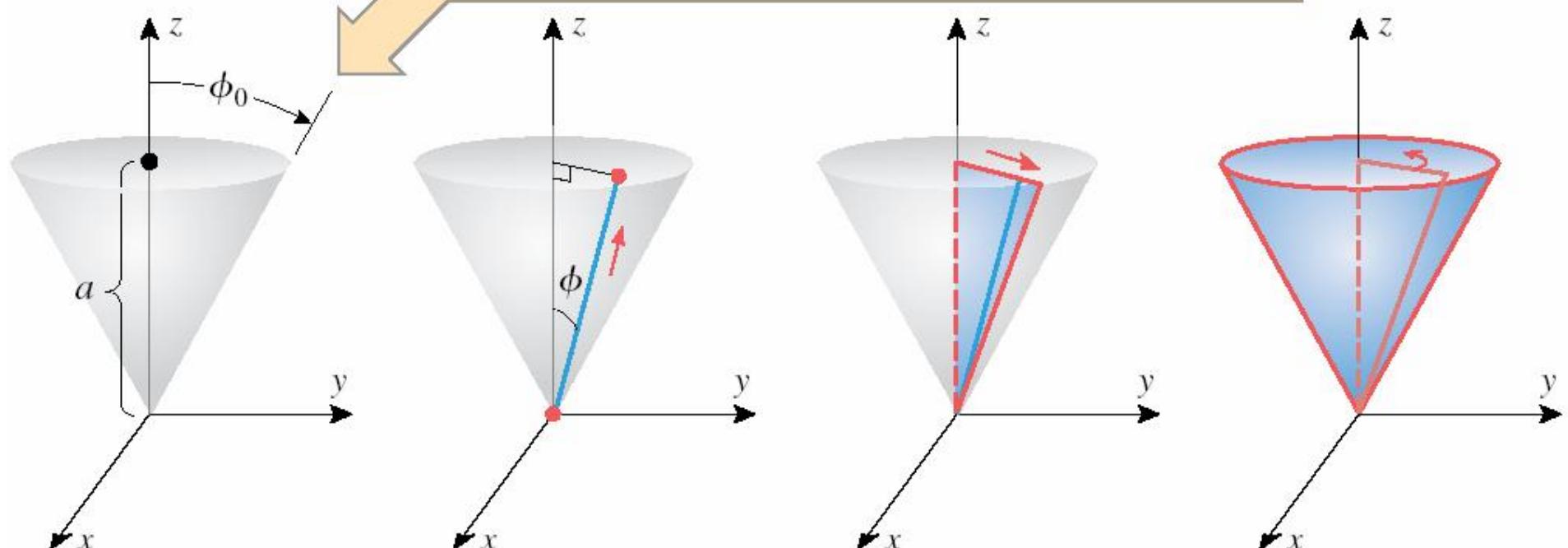
$\rho$  varies from 0 to  $\rho_0$  with  $\theta$  and  $\phi$  held fixed.

$\phi$  varies from  $\phi_1$  to  $\phi_2$  with  $\theta$  held fixed.

$\theta$  varies from 0 to  $2\pi$ .

$$\int_0^{2\pi} \int_0^{\phi_0} \int_0^{a \sec \phi} f(\rho, \theta, \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

This solid is enclosed laterally by the cone  $\phi = \phi_0$  and on top by the horizontal plane  $z = a$ .



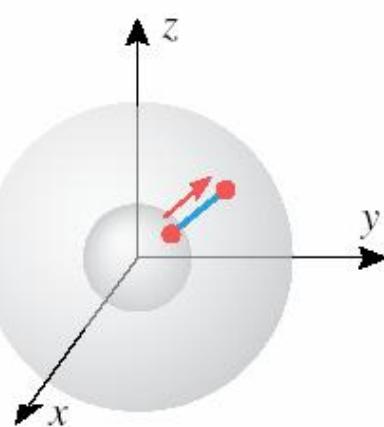
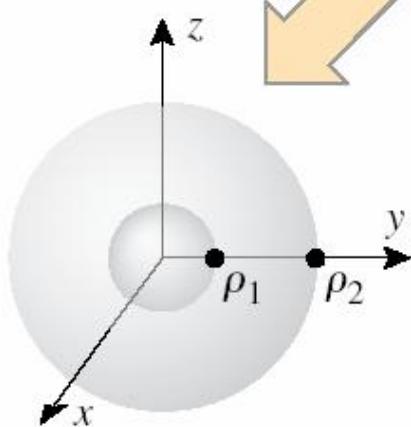
$\rho$  varies from 0 to  $a \sec \phi$  with  $\theta$  and  $\phi$  held fixed.

$\phi$  varies from 0 to  $\phi_0$  with  $\theta$  held fixed.

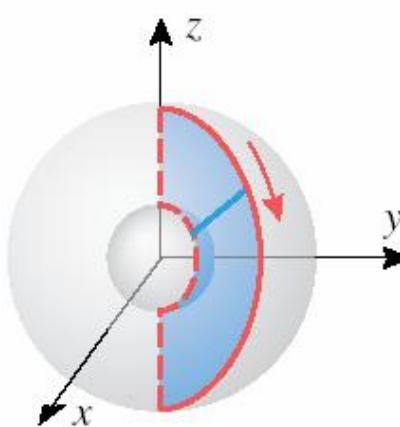
$\theta$  varies from 0 to  $2\pi$ .

$$\int_0^{2\pi} \int_0^{\pi} \int_{\rho_1}^{\rho_2} f(\rho, \theta, \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

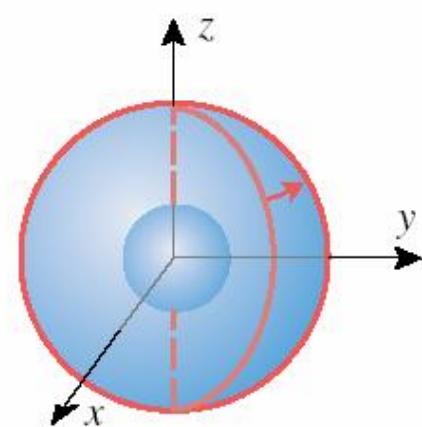
This solid is enclosed between two concentric spheres,  $\rho = \rho_1$  and  $\rho = \rho_2$ .



$\rho$  varies from  $\rho_1$  to  $\rho_2$  with  $\theta$  and  $\phi$  held fixed.



$\phi$  varies from 0 to  $\pi$  with  $\theta$  held fixed.

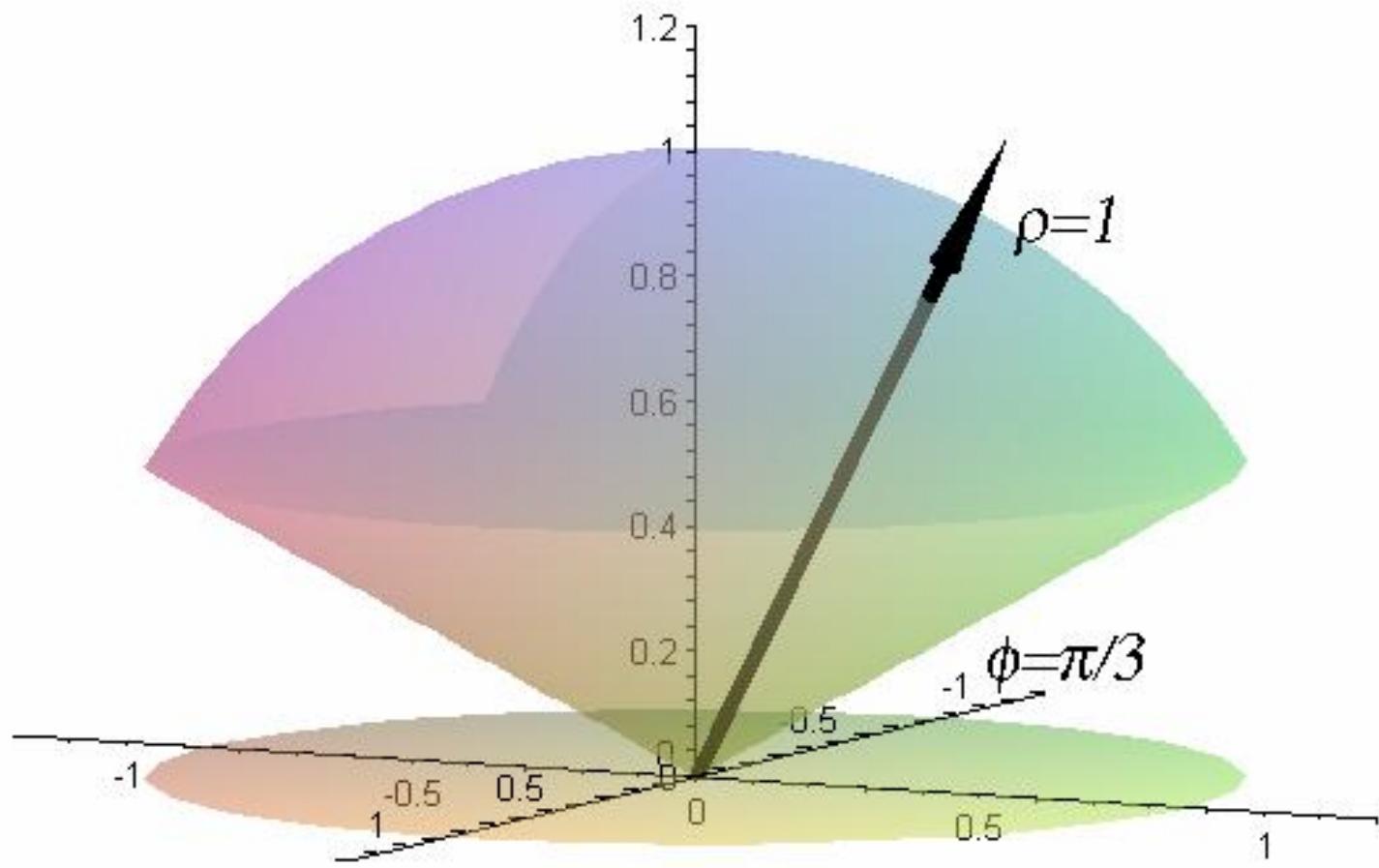


$\theta$  varies from 0 to  $2\pi$ .

$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{a \sec \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \quad (a > 0)$$

จงหาปริมาตรที่ถูกปิดล้อมด้วยทรงกลม  $\rho \leq 1$

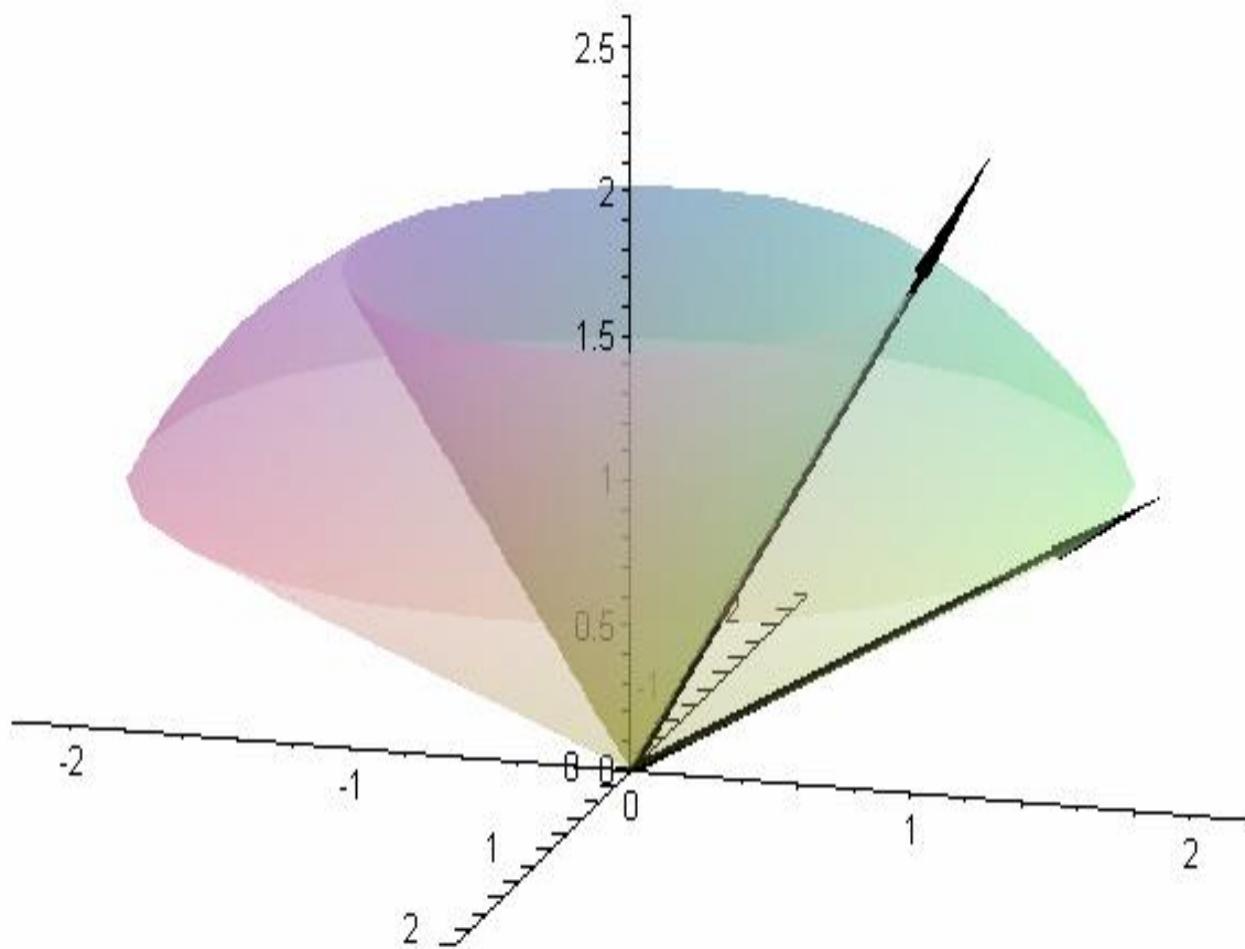
และการวายกลม  $\phi = \frac{\pi}{3}$



จงหาปริมาตรของทรงตันซึ่งอยู่ในอ็อกตาแคน (octant) ที่ 1

ซึ่งถูกปิดล้อมด้วยสมการ

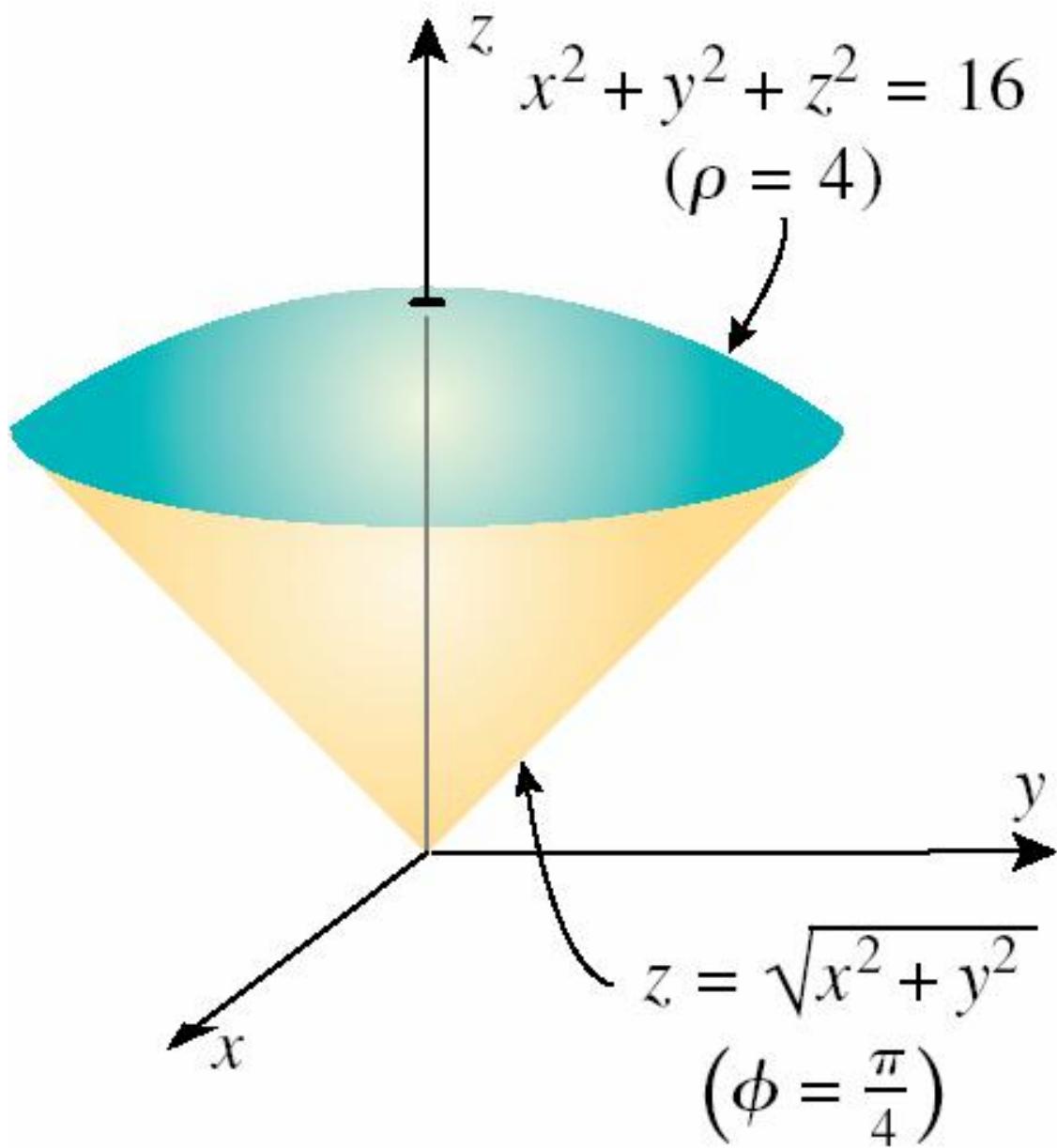
$$\rho = 2 \quad \phi = \frac{\pi}{6} \quad \text{และ} \quad \phi = \frac{\pi}{3}$$



จงหาปริมาตรที่ถูกปิดล้อมด้วยทรงกลม

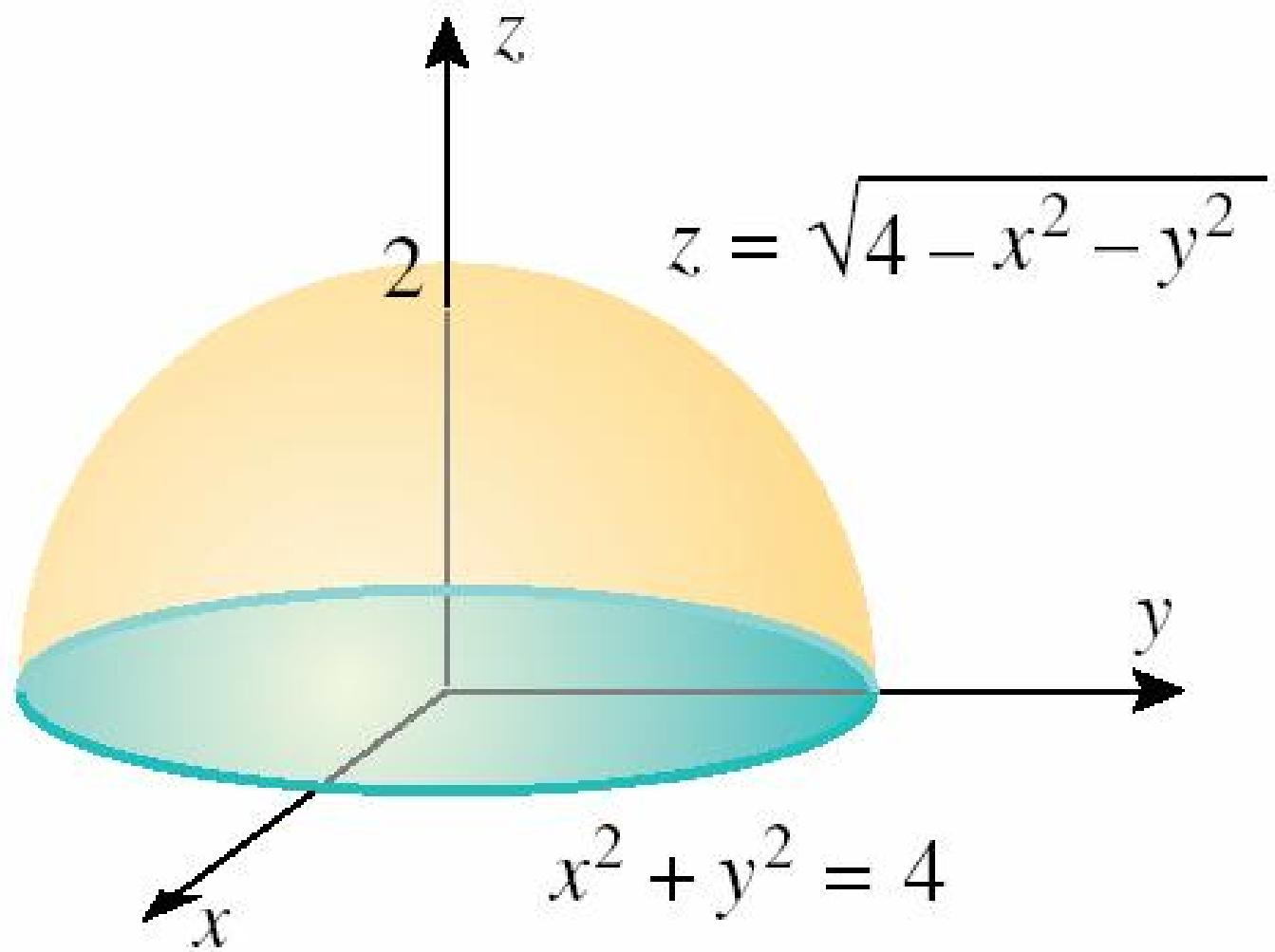
$$x^2 + y^2 + z^2 = 16$$

และรายกลม  $z = \sqrt{x^2 + y^2}$



# ຈົງຫາຄໍາອິນທີກົດ

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2 + y^2 + z^2} dz dy dx$$

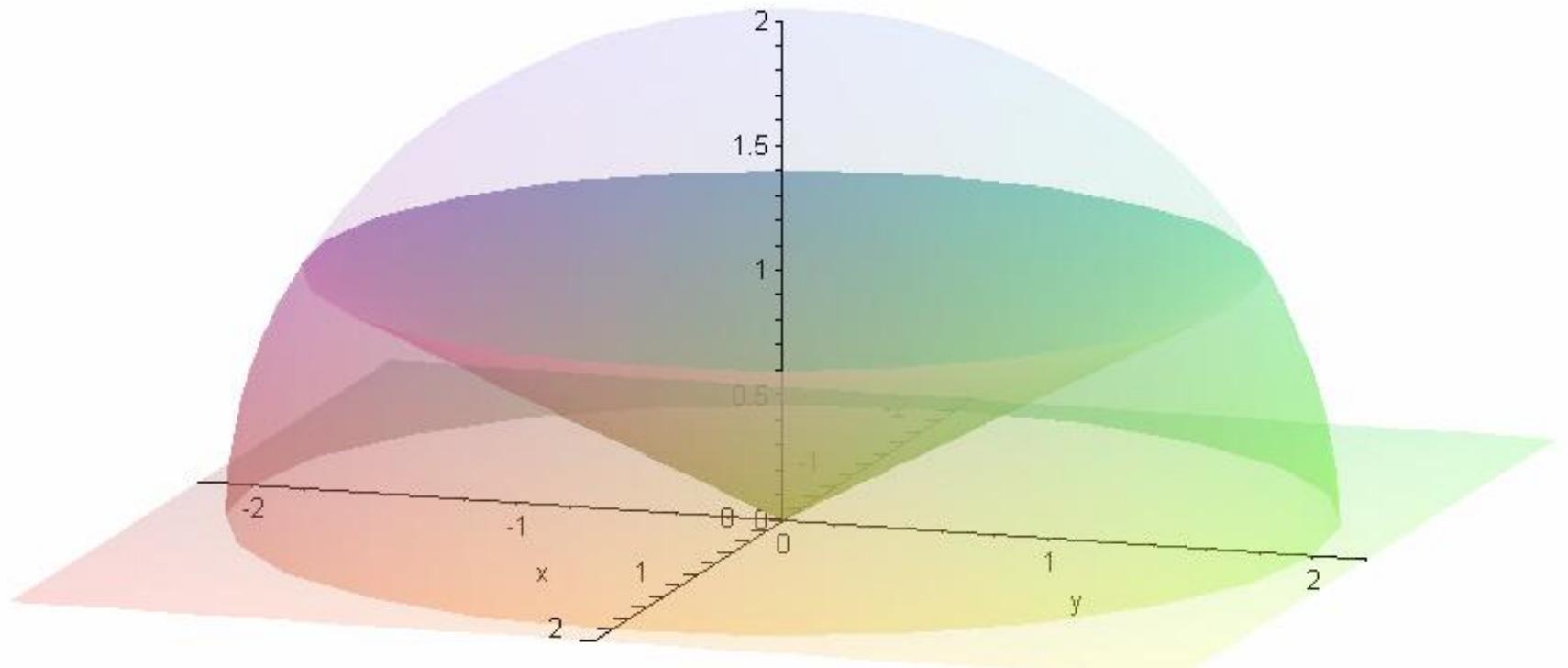


จงหาปริมาตรซึ่งถูกปิดล้อมด้วยทรงกลม

$$x^2 + y^2 + z^2 = 4$$

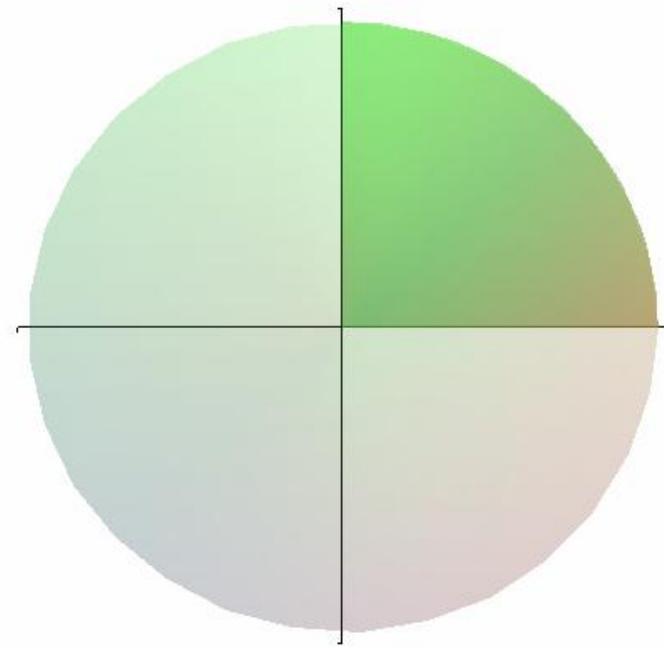
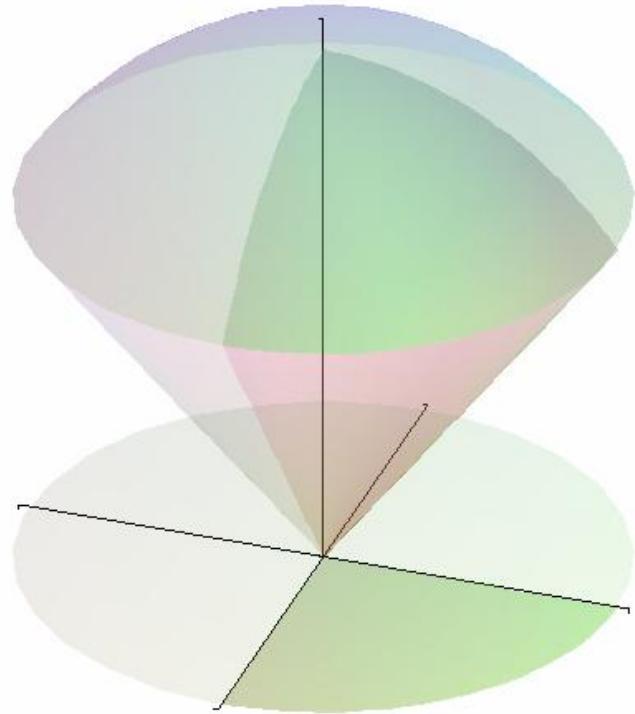
ระนาบ  $z = 0$  และกรวย

$$z = \sqrt{\frac{x^2 + y^2}{3}}$$

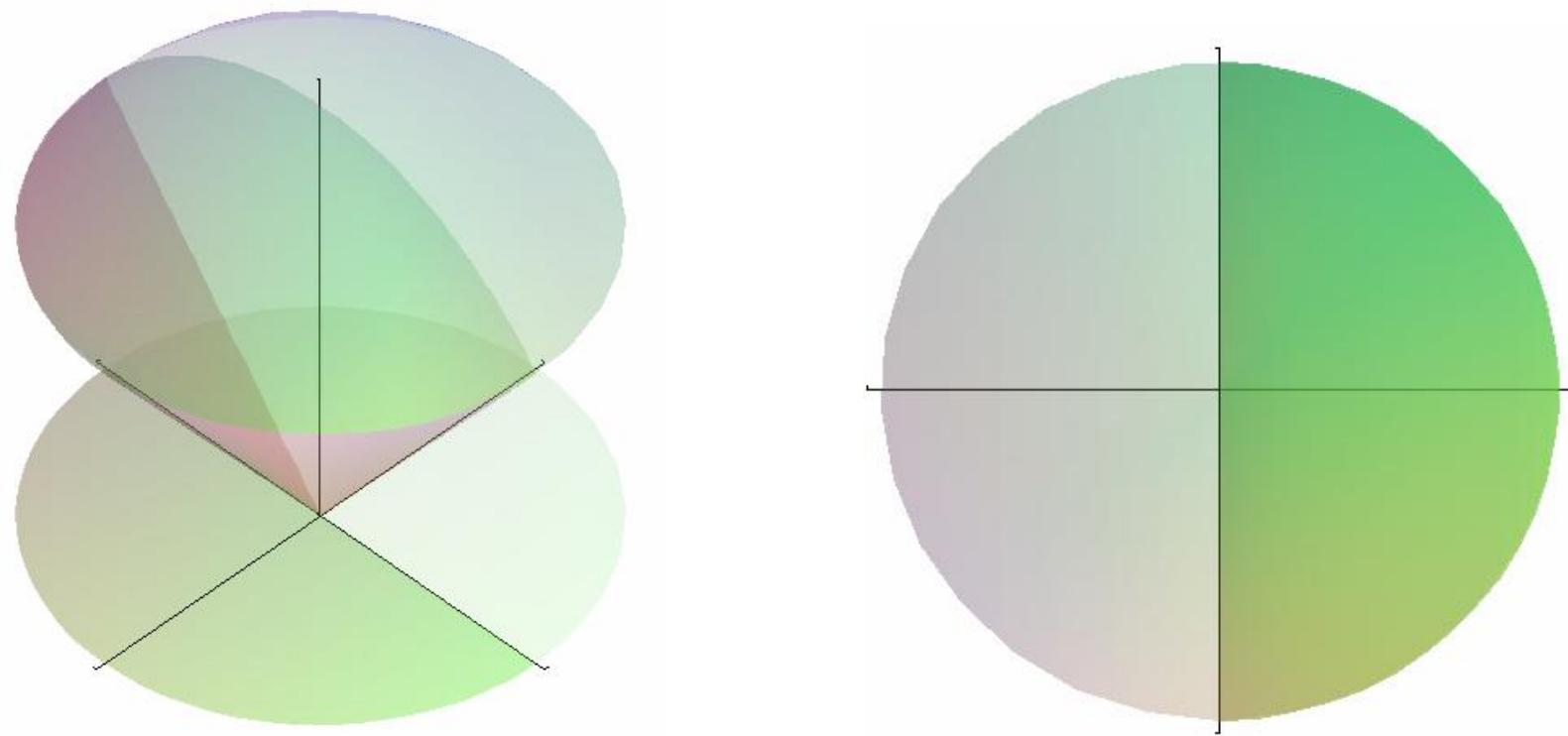


ຈົງຫາຄ່າອິນທີກວ້ລຂອງ

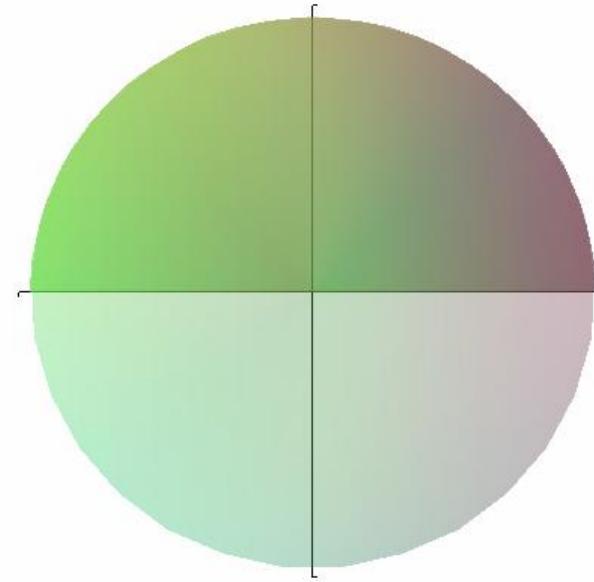
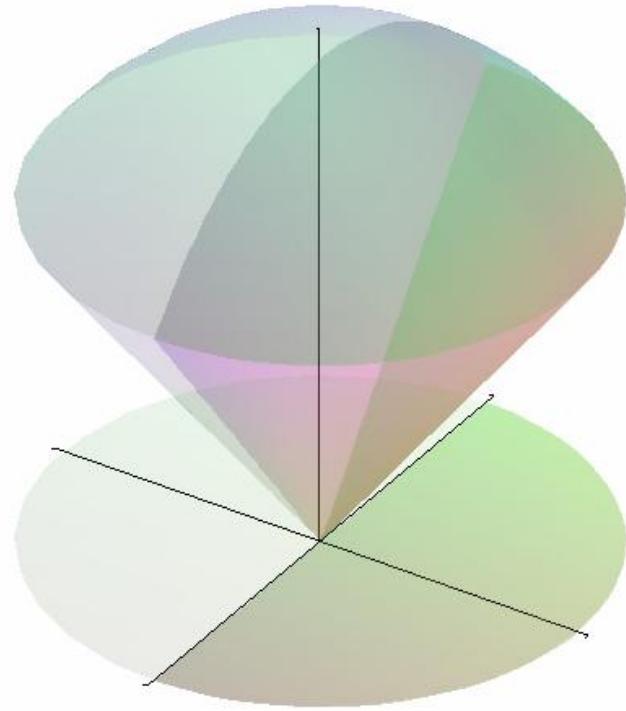
$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z \ dz \ dx \ dy$$



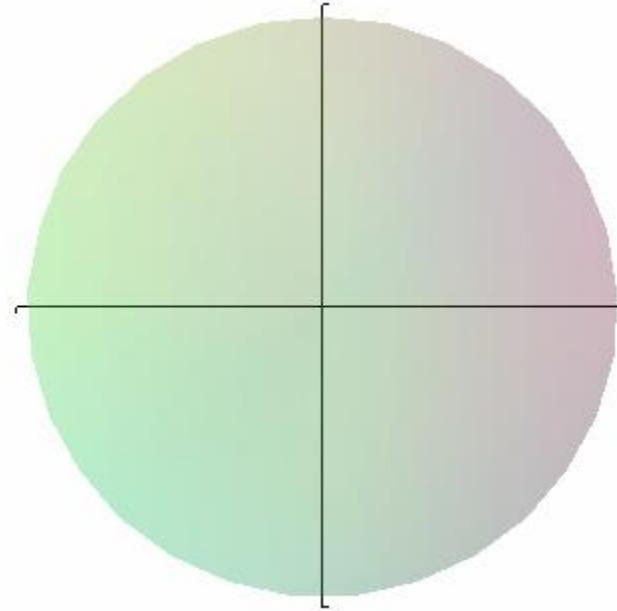
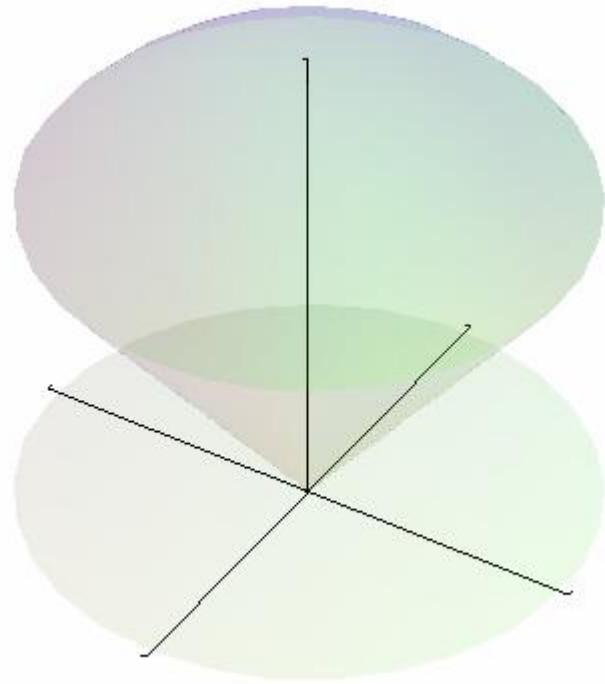
$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z^2 dz dx dy$$



$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z^2 dz dx dy$$



$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z^2 dz dx dy$$



$$\int_{-2}^0 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z^2 dz dy dx$$

จงหาค่าปริพันธ์  $\int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho^4 \sin^2 \phi \cos \phi \, d\rho \, d\phi \, d\theta$

(1)  $\frac{32}{15}\pi$

(2)  $\frac{64}{15}\pi$

(3)  $\frac{72}{15}\pi$

(4)  $32\pi$

(5)  $64\pi$

$$\int_{-3}^0 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} \frac{z}{x^2+y^2+z^2} \ dz \ dy \ dx$$