Analytical and Numerical Methods in Differential Equations (Yanenko 105)

A virtual conference on occasion of the 105th birthday of Academician Nikolai N. Yanenko

March 23-27, 2026

Book of Abstracts

Virtual Host:

School of Mathematics and Geoinformatics
Suranaree University of Technology
Nakhon Ratchasima, Thailand

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Abstract Collection

On the Stochastic Nonlinear Schrödinger Equation

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Let $V(t)$, $t \in [0,T]$, $V(0) = 0$, be a random process with continuous trajectories. The process $V(t)$ is not assumed to be differentiable. The results are also valid if $V(t)$ is a continuous deterministic function. Consider the Cauchy problem:

$$\begin{gathered} i\tilde{u}(x,t) - i\tilde{u}(x,0) + \int_0^t \tilde{u}_{xx}(x,s)ds + \int_0^t f(|\tilde{u}(x,s)|)\tilde{u}(x,s) * dV(s) = 0, \\ \tilde{u}(x,0) = \varphi(x), \quad (x,t) \in \mathbb{R} \times [0,T], \end{gathered} \tag{1}$$

where the last integral is the symmetric integral with respect to the process $V(t)$.

The following theorem holds for the Cauchy problem (1).

Theorem 1. *Let $\varphi(x) \in L^1(\mathbb{R})$, $f(z) \in C^0([0,+\infty))$. The solution to the Cauchy problem (1) is the function*

$$\tilde{u}(x,t) = u(x,t,V(t)) = \frac{1}{(4\pi i t)^{1/2}} \int_{\mathbb{R}} \varphi(y) \exp\left\{ i \left[f(|\varphi(y)|)V(t) + \frac{|x-y|^2}{4t} \right] \right\} dy.$$

The technique of the symmetric integral is used in the work. The symmetric integral was introduced by F.S. Nasyrov [1, 2]. On the one hand, the symmetric integral generalizes the stochastic Stratonovich integral, and on the other hand, it generalizes the Stieltjes integral. The method employed in this work has already been successfully applied by the author to the stochastic nonlinear heat equation [3].

The report will also demonstrate the simulation results.

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Love wave propagation in a multilayered porous system under initial stress with parabolic interfacial irregularity

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This study investigates Love wave propagation in a multilayered geological structure consisting of an isotropic homogeneous layer overlying a fluid-saturated porous medium under initial stress, which rests on a nonhomogeneous elastic half-space with a parabolic interfacial irregularity. The governing equations of motion for each medium are formed using the theory of elasticity by Biot. Analytical solutions are obtained through Fourier transformation techniques. The dispersion relation for the Love waves is derived using perturbation method and Willis' integral formula, revealing the coupled influence of material properties and geometric parameters on wave characteristics. Computational simulations, conducted via MATLAB, demonstrate how the inhomogeneity parameter, anisotropy factor, porosity, initial stress and interface irregularity depth collectively affect phase velocity dispersion. The numerical results indicate that the phase velocity decreases with increasing wave number, inhomogeneity, porosity, and anisotropy. A notable reversal in the phase velocity trend is observed around a critical wave number (1000), which appears to depends on the depth-to-height ratio of the irregularity, in case of the variation in inhomogeneity parameter (from 0 to 3). The results provide new insights into Love wave behavior in complex media, particularly highlighting the significant impact of small-scale interface imperfections that are often neglected in conventional models. These findings advance the theoretical understanding of seismic wave propagation in stratified porous systems and have practical implications for seismic hazard assessment and geotechnical engineering applications.

Wavelet-based numerical approximation in Hölder spaces and application to Riccati differential equations using Vieta–Lucas wavelets

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This article develops a Vieta–Lucas wavelet series method for solving Riccati differential equations. The wavelets are constructed through dilation and translation of orthogonal Vieta–Lucas polynomials. A convergence analysis for the wavelet series is conducted in Hölder-type function classes. By considering partial sums of the wavelet expansion, the study provides error estimates for approximating solutions that belong to the Hölder classes $H_2^{\alpha}[0,1)$ and $H_2^{\phi}[0,1)$, denoted respectively as $E^{(1)}_{2^k,M}(f)$ and $E^{(2)}_{2^k,M}(f)$. The proposed algorithm is applied to both linear and non-linear differential equations, including quadratic Riccati equations. Numerical results obtained via Vieta–Lucas wavelets are compared with those from the MATLAB ODE45 solver. The comparisons demonstrate that the wavelet-based solutions achieve higher accuracy and align closely with exact solutions, confirming the efficiency of the method for Riccati-type problems.

Keywords: Vieta–Lucas wavelet, Hölder class, function approximation, Riccati differential equation, error estimate.

A Sublinear Fractional Sobolev Inequality for Superharmonic Functions

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In this talk, we discuss a Sobolev-type inequality in Lorentz spaces for $\mathcal{L}$-superharmonic functions

$$\|u\|_{L^{\frac{nq}{n-\alpha q},t}(\mathbb{R}^n)} \leq c \left\| \frac{u(x)-u(y)}{|x-y|^{\frac{n}{q}+\alpha}} \right\|_{L^{q,t}(\mathbb{R}^n \times \mathbb{R}^n)}$$

in the sublinear case $p-1<q<1$ and $p-1 \leq t \leq \infty$. The nonlocal nonlinear elliptic operator $\mathcal{L}$ is modeled from the fractional p-Laplacian $(-\Delta_p)^\alpha$ with $0<\alpha<1$ and $1<p<2$. This is based on joint work with Aye Chan May.

High-degree polynomial integrals of geodesic flows and the generalized hodograph method

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In the talk we shall consider integrable geodesic flows on 2-surfaces admitting an additional high-degree polynomial in momenta first integral. Generally speaking, the search for such integrals leads to certain complicated quasi-linear systems of PDEs. As proved in [1], typically these systems turned out to be *semi-Hamiltonian* [2]. In particular, it means that it is possible to apply *the generalized hodograph method* [2] to these systems to construct its solutions. However, the direct implementation of this method turned out to be implicit and very complicated procedure [3], [4], [5].

We present an explicit algorithm based on the generalized hodograph method which allows one to construct many particular solutions to such systems. Explicit examples of Riemannian metrics and polynomial first integrals of degrees 3, 4 and 5 are also provided [6].

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Modeling and Solution of Differential Equation for Cracked Nonhomogeneous Nanobeams

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Nonhomogeneous nanobeams are extensively used in advanced aerospace, mechanical, and biomedical systems, where the presence of cracks can significantly influence their vibration behavior. Accurate modeling of such effects is therefore essential for reliable structural design and damage assessment. In this work, an analytical formulation is presented for the free vibration analysis of cracked nonhomogeneous nanobeams resting on Winkler-Pasternak elastic foundations. The non-homogeneity in material properties are considered along the thickness direction according to a specific mathematical function. Each crack is represented by a rotational spring that partitions the nanobeam into multiple sub-beams. Using Euler-Bernoulli beam theory combined with Eringen's nonlocal elasticity, a governing partial differential equation is derived to analyze the vibrational behavior. By enforcing appropriate continuity and boundary conditions, frequency values are obtained. The proposed model is validated through comparison with benchmark solutions in special cases. Parametric studies are conducted to investigate the effects of gradient index, nonlocal parameter, foundation stiffness, and crack characteristics on the first four natural frequencies and corresponding mode shapes. The developed analytical PDE-based framework offers an effective tool for predicting vibration characteristics of cracked nonhomogeneous nanostructures.

Fundamental solutions of a transversely isotropic elastic medium equations

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Linear differential equations of the fourth order

$$L_1 u \equiv u_{xxxx} + 2u_{xxyy} + u_{yyyy} + B_1(u_{xxzz} + u_{yyzz}) + B_2 u_{zzzz} = 0, \tag{1}$$

$$L_2 u \equiv B_3(u_{xxxx} + 2u_{xxyy} + u_{yyyy}) + B_4(u_{xxzz} + u_{yyzz}) + u_{zzzz} = 0 \tag{2}$$

were considered. Here B_1, B_2, B_3, B_4 are positive constants characterizing a transversely isotropic elastic medium [1].

The fundamental solutions of equations (1), (2) are solutions of the equations

$$L_1 u = \delta(x)\delta(y)\delta(z), \tag{3}$$

$$L_2 u = \delta(x)\delta(y)\delta(z). \tag{4}$$

By stretching transformations, equations (1), (3) and (2), (4) can be reduced to equations

$$Lu \equiv u_{xxxx} + 2u_{xxyy} + u_{yyyy} + b(u_{xxzz} + u_{yyzz}) + u_{zzzz} = 0, \tag{5}$$

$$Lu = \delta(x)\delta(y)\delta(z). \tag{6}$$

Here $b = B_1/\sqrt{B_2}$ or $b = B_4/\sqrt{B_3}$.

Using the symmetry finding algorithm [2], the symmetries of the equation (5) were found.

The symmetries of the equation (6) were found. To find the symmetries of of the equation (6)) the results of [3] were used. The invariant under this symmetry operators solution of the equation (6) was found. The invariant fundamental solution is expressed in elementary functions.

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Numerical Study of Flutter in an Orthotropic Rectangular Plate with Mixed Boundary Conditions

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In this paper, we consider solutions to the problem of aeroelastic oscillations in an orthotropic plate with mixed boundary conditions. A new algorithm without saturation is proposed, which is based on the idea of numerical algorithms without saturation introduced by Konstantin Ivanovich Babenko. The algorithm uses polynomial interpolation, which has the property of being more accurate for smoother functions. Since the solution is smooth within the rectangle, the algorithm takes advantage of this smoothness and responds appropriately. It is not necessary to know the exact smoothness a priori, as the algorithm will automatically adapt to it. The interpolation points are chosen based on the roots of the Chebyshev polynomial, which are concentrated near the corners of the rectangle where the solution has singularities.

To derive a discrete form of the problem, a bi-harmonic operator is applied to the given interpolation formula. This means that the formula is differentiated twice or four times with respect to both x and y coordinates. Experimental results indicate that if the number of grid points in the x and y directions is less than 50, the discretization process does not result in a significant accumulation of errors.

Thus, the main task is to construct an interpolation formula that satisfies the mixed boundary conditions. A number of numerical examples and computational experiments have been presented. New mechanical phenomena have been identified.

The authors gratefully acknowledge financial support by IPMeh RAS No.124012500437-9.

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The use of conservation laws for the analytical solution of the elastic problem of antiplane deformation

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The authors have constructed an infinite series of conservation laws when the conserved current linearly depends on the components of the voltage tensor. Using these conservation laws, the authors constructed an analytical solution to the problem of antiplane deformation.
The formulation and one of the solutions using complex variables on antiplane deformation are described, for example, in the book [1].
This task has the form

$$\partial_x \tau_{xz} + \partial_y \tau_{yz} = 0, \quad \partial_y \tau_{xz} - \partial_y \tau_{xz} = 0. \tag{1}$$

The boundary conditions have the form

$$n_1 \tau_{xz} + n_2 \tau_{yz}|_\Gamma = T(s), \quad \oint\limits_\Gamma T(s) ds = 0. \tag{2}$$

Here (τ_{xz}, τ_{yz}) are the components of the stress tensor, (n_1, n_2) - the vector of the external normal to the contour Γ, bounding the finite region S, $T(s)$ - the specified external loads on the external contour.

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Integration of PDEs via Differential Constraints and $\mathcal{C}^\infty$-structures

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Following Yanenko's classical method of differential constraints [1] and the geometric reinterpretation in jet space by Olver and Rosenau [2], we present a procedure for integrating PDEs. The approach consists of adjoining compatible auxiliary differential equations to reduce a PDE to a finite type Vessiot distribution that becomes involutive on an open set. Classical solvable structures [3] have been used to integrate such distributions [4], but they are restrictive and difficult to construct. We propose employing $\mathcal{C}^\infty$-structures of distributions, a generalization of solvable structures with relaxed bracket conditions, to ease their identification and construction [5].

Given a $\mathcal{C}^\infty$-structure, that is, an ordered sequence of vector fields $\{X_1, X_2, X_3\}$ where each X_i is a $\mathcal{C}^\infty$-symmetry of successively expanded distributions generated by the total derivative vector fields A_x, A_t and the preceding fields, one obtains a systematic parametrization of integral manifolds of the Vessiot distribution through solving a sequence of three completely integrable Pfaffian equations.

We illustrate the method with explicit examples.

Keywords: differential constraints, Vessiot distributions, involutive systems, $\mathcal{C}^\infty$-structures, Pfaffian equations, exact solutions.

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The direct and inverse problems for the Kelvin-Voigt equations. Application to incompressible inhomogeneous active fluids.

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In the present report, we study the Kelvin-Voigt equations with a singular source term:

$$\begin{cases} \partial_t \rho_n + \operatorname{div}(\rho_n \mathbf{v}_n) = 0 \text{ in } Q_T, \\ \partial_t(\rho_n \mathbf{v}_n) + \mathbf{div}(\rho_n \mathbf{v}_n \otimes \mathbf{v}_n) = -\nabla \pi + \Delta \mathbf{v} + \Delta \partial_t \mathbf{v} + \varphi_n(t)\mathbf{f} \text{ in } Q_T, \\ \operatorname{div} \mathbf{v} = 0 \text{ in } Q_T, \\ \mathbf{v}(x,0) = \mathbf{v}_0(x) \text{ in } \Omega, \\ \mathbf{v}(x,t) = \mathbf{0} \text{ on } \Gamma_T, \end{cases} \tag{1}$$

where ρ_n is the unknown density, $\mathbf{v}_n$ is the unknown velocity field, π_n is the unknown scalar-valued function, $\varphi_n(t)$ is the unknown source term, $\mathbf{f} \in L^\infty(0,T;\mathbb{V}') = L^\infty(0,T;(W^{-1,2}_{\text{div}}(\Omega))^d)$ is a given functional, $Q_T = \Omega \times (0,T)$, Ω is a bounded domain of $\mathbb{R}^d$, $d \in \{2,3\}$, and T is a given positive constant, and $\Gamma_T = \partial\Omega \times (0,T)$, with $\partial\Omega$ denoting the boundary of Ω.

- In the **direct problem** the unknown functions are $(\mathbf{v}_n, \pi_n, \rho_n)$. Here, the source term $\varphi_n(t)$ approximates the Dirac delta function $\delta_{(t=0)}$. Such the direct problem was studied in [1] for $\mathbf{f} = \rho_n \mathbf{v}_n$.
- In the **inverse problem**, the source term is also unknown. We introduce the integral overdetermination condition
$$\int_\Omega \rho_n \mathbf{v}_n \cdot \boldsymbol{\omega}\, dx = j_n(t),$$
where $j_n(t)$ approximates the discontinuous function at $t = 0$. Similarly with item 1, the inverse problem for $\mathbf{f}(x,t) \equiv \boldsymbol{\omega}(x)$ was studied in [2].

In both tasks, the basic idea is to pass to the limit from $n \to \infty$. An infinitesimal initial layer is introduced by applying the rescaling procedure $\vartheta = nt : [0, 1/n] \mapsto [0,1]$. In the limit, the velocity has a gap at $t = 0$. This means that a new initial velocity is accepted. Since there is no singular source term in the mass balance, the density does not have a gap at $t = 0$ in the limit as $n \to \infty$.

The functional $\mathbf{f}_N = \sum_{i=1}^{N} \alpha_i(t)\delta_{(x=x_i(t))}$ describes N-self-propelling particles in active fluids with N trajectories $x = x_i(t)$. It is important to note that $\mathbf{f}_N$ is not from $L^\infty(0,T;(W^{-1,2}_{\text{div}}(\Omega))^d)$, but from $L^\infty(0,T;(W^{-s,2}_{\text{div}}(\Omega))^d)$, such that $(W^{s,2}_{\text{div}})(\Omega))^d \subset C(\Omega)$, but this can be revised by applying the Hahn-Banach theorem. The presence of $\varphi_n(t)$ is linked with spontaneous flows when self-propelling particles start moving abruptly in one direction and create such a flow.

Actually, following the control theory [3], such a functional $\varphi_n(t)\mathbf{f}_N$ can be found by minimising the functional
$$\|\mathbf{v}_n(\cdot, 1/n) - \mathbf{u}(\cdot)\|_{2,\Omega} \to \inf,$$
where $\mathbf{u}(\cdot)$ is the trace at $t = 1/n$, charcterising the spontaneous flow.

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Integrability of (1+2)-dimensional Kudryashov-Sinelshchikov (KS) equation using Painlevé Analysis and their exact solutions

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In this research work, we consider a nonlinear fourth-order (1+2)-dimensional Kudryashov-Sinelshchikov (KS) equation which represents the wave propagation of pressures in liquids that contain gas bubbles. A direct Integrability of the KS equation is analysed using Painlevé Analysis with Singular Manifold Method (SMM). With the help of WTC algorithm, we show that the (1+2) KS equation is Painlevé integrable. Then by truncating the Painlevé expansion we obtain the Auto-Bäcklund Transformation (ABT). By taking suitable forms of Manifold, various exact solutions based on the obtained Auto-Bäcklund transformation are derived. The consistancy check for these solutions are also performed. Representative solutions are presented in the from of 2D and 3D plots to understand the geometric perspective of the solutions.

The authors gratefully acknowledge financial support by Anusandhan National Research Foundation (ANRF), India under the Core Research Grant with File No: CRG/2023/005418, dated 20 August, 2024 and DST-FIST grant SR/FST/MS-I/2024/173 to the Department of Mathematics, Pondicherry University, Puducherry, India.

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The influence of surface elasticity on shear wave propagation in a cylindrical layer structure with an imperfect interface

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This study explores the influence of surface elasticity on the propagation characteristics of shear acoustic waves in a cylindrical layered structure. The structure consists of a functionally graded piezoelectric (FGPE) layer imperfectly bonded to a concentric fiber-reinforced composite (FRC) cylindrical substrate. The material properties of the piezoelectric layer are assumed to vary continuously along the radial direction according to a prescribed functional gradient. A special function approach is employed to derive the dispersion relations under electrically short-circuit and open-circuit boundary conditions. Numerical simulations are performed, and the results are presented graphically to illustrate the effects of various parameters, including the radius ratio of the concentric cylinder, the functional gradient parameter, and the imperfect bonding parameter. The findings of this theoretical investigation are expected to be beneficial for enhancing the performance of piezocables, actuators, and surface acoustic wave sensors.

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Autonomous and Nonautonomous Dynamics of an SIRS Model: An Application to Seasonal Influenza in the Congo

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This study develops and analyzes an SIRS epidemic model with convex incidence and saturated treatment under both autonomous and nonautonomous frameworks. For the autonomous system, we characterize the disease-free and endemic equilibria and perform a detailed bifurcation analysis, revealing backward and saddle-node bifurcations, as well as Hopf bifurcations that generate endemic bubbles. Furthermore, the bifurcation structure uncovers a codimension-two double-zero bifurcation arising from the interaction between saddle-node and Hopf bifurcations. The nonautonomous extension incorporates seasonal variations in transmission and recovery rates, capturing realistic periodic forcing observed in infectious diseases such as influenza. Using epidemiological data from the Democratic Republic of the Congo, we identify December as the peak influenza season. Analytical results establish conditions for the existence and global stability of a positive periodic solution, while numerical simulations demonstrate that seasonality can induce complex dynamics, including multiperiodic and chaotic oscillations. Low seasonal intensity sustains disease coexistence, whereas strong seasonal forcing may lead to population extinction. The emergence of quasiperiodic (torus) and chaotic (strange) attractors highlights how seasonal forcing can transform regular epidemic cycles into irregular outbreaks, providing new insights into the role of seasonality in infectious disease dynamics and control [1].

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Hybrid finite difference WENO schemes for Ten-Moment Gaussian closure equations with source term

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A hybrid weighted essentially non-oscillatory (WENO) finite difference scheme is proposed for computing discontinuous solutions of the ten-moment Gaussian closure equations. A salient feature of the proposed scheme is the use of low-cost component-wise reconstruction of the numerical fluxes in smooth regions and non-oscillatory characteristic-wise reconstruction in the vicinity of discontinuities. A troubled-cell indicator which measures the smoothness of the solution, and built on utilizing the smoothness indicators of the underlying WENO scheme, is employed to effectively switch between the two reconstructions. The resulting hybrid WENO scheme is simple and efficient, is independent of the order and type of the WENO reconstruction, and it can be used as an effective platform to construct finite difference schemes of any arbitrary high-order accuracy. For demonstration, we have considered the fifth order WENO-Z reconstruc- tion. We have performed several 1D and 2D numerical experiments to illustrate the efficiency of the proposed hybrid algorithm and its performance compared to the standard WENO-Z scheme. Numerical case studies shows that the present algorithm achieves fifth order accuracy for smooth problems, resolves discontinuities in a non-oscillatory manner and takes 25less computational time than the WENO-Z scheme while retaining many of its advantages. The authors gratefully acknowledge financial support by SERB-DST CRG/2021/004078

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Dynamics and pattern formation in a diffusive predator-prey model with fear effect

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In this paper, we investigate the dynamical behavior and positive steady states of a diffusive predator–prey system that incorporates a fear effect and a Beddington–DeAngelis functional response under Neumann boundary conditions. We analyze the qualitative properties of timedependent solutions and examine diffusion-driven instabilities leading to the formation of stationary spatial patterns (Turing patterns).

Finite Energy Solutions to Sublinear Elliptic Systems Involving Measure

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We characterize the existence of a pair of positive finite energy solutions to the homogeneous elliptic systems

$$\begin{cases} (-\Delta)^\alpha u = \sigma v^{q_1} \quad \text{in} \quad \mathbb{R}^n, \\ (-\Delta)^\alpha v = \sigma u^{q_2} \quad \text{in} \quad \mathbb{R}^n, \\ \liminf\limits_{|x|\to\infty} u(x) = 0, \liminf\limits_{|x|\to\infty} v(x) = 0 \end{cases}$$

in the sublinear case $0 < q_1, q_2 < 1$. Here σ is positive Borel measure on $\mathbb{R}^n$, and $(-\Delta)^\alpha$ is the fractional Laplace operator with $0 < \alpha < \frac{n}{2}$. In addition, we obtain parallel results for the corresponding system involving the classical Laplace operator $-\Delta$ on an arbitrary domain $\Omega \subset \mathbb{R}^n$ with positive Green's function.

This work is supported by the Thammasat University Research Unit in Gait Analysis and Intelligent Technology (GaitTech). A.C.M. gratefully acknowledges financial support from the Excellent Foreign Student (EFS) scholarship, Sirindhorn International Institute of Technology (SIIT), Thammasat University.

Closure Conditions for the Governing Equations of Steady-State Evaporative Convection in Binary Mixtures

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The work addresses the closure of stationary problems of evaporative convection in a system of two binary mixtures filling a narrow, elongated horizontal channel. The mathematical model is based on the Navier-Stokes equations and the balance relationships for heat and mass transfer. The model incorporates terms accounting for second-order effects: thermal diffusion (Soret effect) and diffusion thermoeffect (Dufour effect). They are added to the model as regular contributions, meaning their inclusion or exclusion does not alter the fundamental structure (or type) of the equations. This allows for their parametric analysis within the model framework: they can be retained where their influence is significant and neglected where their impact is negligible, without introducing singularities [1].

In accordance with the experimental results of [2], the flow can be regarded as unidirectional. Given this assumption, a closed-form solution of the governing equations can be constructed by considering the temperature and concentration functions as polynomial of the longitudinal coordinate x. In particular, a linear dependence on x yields an exact solution belonging to the well-established class of Ostroumov–Birikh solutions, which has proven effective in modeling flows with weak evaporation [3].

A critical aspect of conjugate problems is the formulation of boundary conditions and closure relations, both at the external boundaries of the flow domain and at the internal interface. It is important to note that accounting for the inhomogeneity of the evaporating liquid introduces several possibilities for formulating boundary conditions for the concentration function. In addition to the equality of mass fluxes at the interface, one may consider a Henry's law equilibrium condition or/and specify the saturated vapor concentration. Furthermore, it is necessary to consider impact of both thermocapillary and solutocapillary forces as well as their influence on the flow characteristics. It should be noted that while the dependence of surface tension on temperature is often linear for many liquids, its dependence on concentration remains linear only within a narrow range of the temperature and concentration variations.

This presentation proposes a hierarchy of problem formulations for modeling weak evaporation. It covers four types of boundary conditions for the temperature field at the domain boundaries, accounts for both linear and quadratic dependencies of surface tension on the concentration of the volatile component, and examines various closure relations such as specifying the gas flow rate in the upper layer or the liquid flow rate in the lower layer. Furthermore, the presentation addresses open questions in modeling conjugate flows such as the violation of the discontinuity of velocity at the interface in real evaporative convection processes.

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Group approach to the problem of the connecting kinetic equations to the continuous medium equations in the one-dimensional case: from the idea of closing the momentum system to the group foliation method

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The problem of deriving continuum media equations from kinetic ones using methods of group analysis is discussed.

The original idea was a modification of the classical approach and consisted in obtaining continuum media equations by truncating the infinite hierarchy of moment equations and closing it with certain relations. The key distinction of our approach from previous ones was that the closure relations must be invariant under the symmetry group of the original kinetic equation.

This idea was tested on the simplest one-dimensional kinetic equation

$$f_t + cf_x + (Ff)_c = 0. \tag{1}$$

Group analysis of equation (1) is performed within the class of diffeomorphisms of the space of all variables t, x, c, f (and additionally F in the case of an equivalence group) satisfying the following three conditions:

- invariance of the relations
$$dx = c\,dt, \qquad dc = F\,dt \tag{2}$$
which expresses the preservation of the physical relationship among the quantities (t, x, c, F);
- invariance of the family of lines
$$dx = dt = 0 \tag{3}$$
required to preserve the physical meaning of moment variables;
- invariance under changes of variables of the quantity
$$(1 + c\theta_x + F\theta_c)f(t, x, c)\,dx\,dc \tag{4}$$
on any surface $t = \theta(x, c)$ expressing the independence of the number of particles from the choice of coordinate system.

We established that the group of point transformations of the space of variables (t, x, c, f, F) preserving relations (2), (3) and the quantity (4) coincides with the group of diffeomorphisms of the (t, x)-space and the induced transformations of the remaining variables; the equivalence group of equation (1) coincides with this group.

A group classification of equations (1) within the specified class of transformations has been carried out. For the obtained symmetry groups, their action on moment variables has been computed and the corresponding invariants have been found.

In the case $F = 0$, the original idea was fully realized: the derived differential invariant led to the system

$$\rho_t + (\rho u)_x = 0, \qquad u_t + uu_x = 0,$$

which is well known as the “pressureless hydrodynamics” equations. Each solution $(\rho(t, x), u(t, x))$ corresponds to the distribution

$$f(t, x, c) = \rho(t, x)\,\delta(c - u(t, x)).$$

For equations possessing three-dimensional (submaximal) symmetry groups, the original formulation of the problem had to be radically transformed. Indeed, for such equations it turned out that moment variables of order higher than the first do not just exist because the corresponding integrals diverge. The only remaining momentum equation usually considered as the continuity equation contains sources and sinks; that contradict the original goal of deriving continuum media equations from the moment hierarchy.

These difficulties and the resulting paradoxes require a reformulation of the problem. It turned out that a formulation essentially equivalent to the original one can be expressed directly in terms of the distribution function f. Specifically, one must solve simultaneously the kinetic equation (1) and the equations expressing the invariant representation of f through the first two moment variables. The evolution equations for density and flow velocity which naturally qualify as "continuum equations" — then arise as compatibility conditions for this combined system.

It should be noted that the last remaining moment equation appeared to be a consequence of these compatibility conditions. This makes the consideration of moment equations useless at all.

Finally we observe that the resulting scheme for solving the problem is closely related to the well-known method of group foliation, with two distinctions. The first is that the integral (rather than purely differential) quantities are introduced; the second one is that we consider not all solutions of the kinetic equation but only a specific subset thereof.

Mathematical Modeling of some Cerebral Vascular Pathologies

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Cerebral arteriovenous malformation (AVM) is a congenital vascular pathology in which the arterial and venous beds are connected by a tangle of fused pathological vessels. A common and preferred method of treating it is neurosurgical embolization. In this case, pathological vessels are filled with a special hardening substance (embolic agent).

For mathematical modeling of embolization, taking into account changes in blood flow in the environment of the pathology, a two-phase filtration model is used, coupled with a hydraulic model of the vessels surrounding the pathology. Mathematically, the model is described by a chain of integro-differential hyperbolic equations with a non-convex flow function.

The embolization problem is considered as an optimal control problem, in which the control is the dependence of the embolization supply on time, and the integral target functionality and constraints are selected for medical reasons. The stated optimal control problem is studied based on clinical data on blood flow near the pathology obtained during neurosurgical operations.

Internal waves in a thermally stratified reservoir: modeling and field observations in South Baikal

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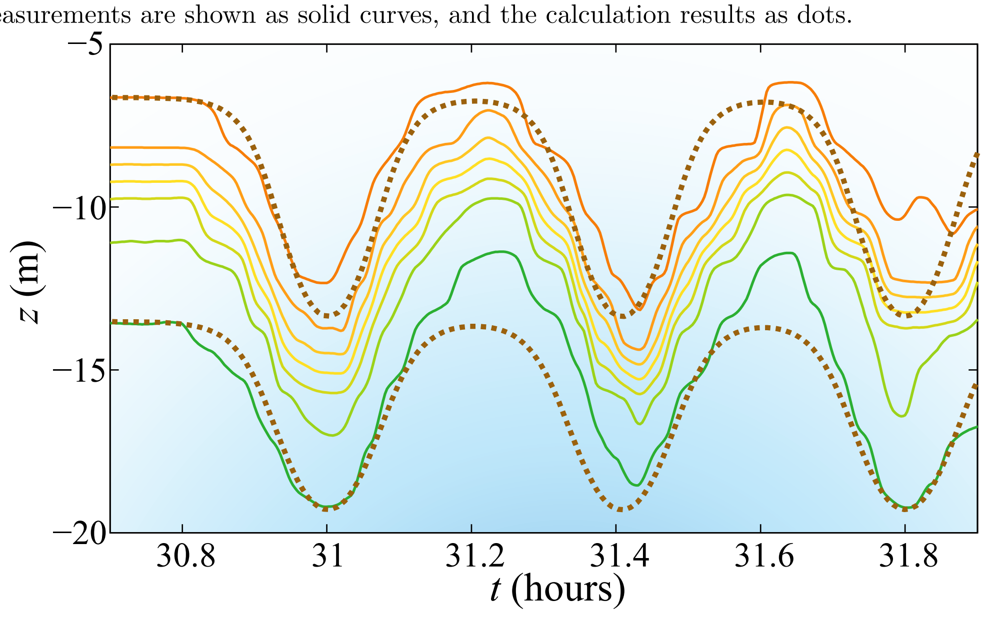
We apply nonlinear multilayer shallow water equations in the Boussinesq approximation to describe the evolution of large-amplitude internal waves in a closed thermally stratified reservoir. The model allows to take into account the fine structure of stratification, velocity shear, friction at interfaces, and non-hydrostatic pressure distribution in the upper and/or lower layers [1, 2, 3]. Quasi-periodic solutions in the class of traveling waves are constructed, and the influence of the velocity shear in layers on the profile of a solitary wave is demonstrated. Test calculations of wind-driven internal waves and seiche oscillations in a large stratified reservoir are performed. It has been shown that moderate wind action can induce internal waves of fairly large amplitude. The results of processing data from field observations on temperature fluctuations in the surface layer of South Baikal in the autumn-summer period are presented. The characteristic temperature fluctuations at different horizons are demonstrated, obtained both using free-floating buoys at a distance from the shore and using a moored buoy station in coastal waters. Presenting these data as isotherms allows the proposed multilayer model to be used to interpret field observations. In particular, quasi-periodic wave structures and internal solitary waves are identified and modeled. A typical example of a quasi-periodic wave packet is shown in the figure, where the data from field measurements are shown as solid curves, and the calculation results as dots.



The authors gratefully acknowledge financial support by the Ministry of Science and Higher Education of the Russian Federation, the grant No. 075-15-2024-533.

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Concept of Index and Some Structural Properties of Partial Differential-Algebraic Equations

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We consider an evolutionary system of partial differential equations

$$\Lambda_k(D_t, D_x)u := \sum_{j=0}^{k} \mathcal{A}_j(D_x)D_t^j u = f(x,t), \tag{1}$$

where $(x,t) \in \mathbf{U} = X \times T$, $T = [\alpha, \beta]$, $X = [x_0, x_1]$, $\mathcal{A}_j(D_x) = \sum_{i=0}^{k_j} A_{j,i}(x,t)D_x^i$, $D_t \equiv \partial/\partial t$, $D_x \equiv \partial/\partial x$, $A_{j,i}(x,t)$ are $(n \times n)$ matrices, $f(x,t)$, $u \equiv u(x,t)$ are the given and the unknown vector-functions, respectively, $D_t^0 u = u$, $D_x^0 u = u$. In stating the problem, we follow [1] and employ techniques from [2]. It is assumed that in system (1) the operator at the leading term possesses the following property: on any restriction of the domain $\tilde{\mathbf{U}} \subset \mathbf{U}$ there exists a free term $\varphi(x,t)$ such that the system of equations

$$\mathcal{A}_k(D_x)z = \varphi(x,t) \tag{2}$$

has no solutions. It is common to refer to such systems as partial differential-algebraic equations (PDAEs). In particular, for first-order systems

$$A(x,t)D_t u + B(x,t)D_x u + C(x,t)u = f(x,t),$$

where $A(x,t)$, $B(x,t)$, $C(x,t)$ are $(n \times n)$-matrices, $\mathcal{A}_1(x,t,D_x) = A(x,t)D_x^0$, $\mathcal{A}_0(x,t,D_,x) = B(x,t)D_x + C(x,t)$, condition (2) is equivalent to

$$\det\ A(x,t) = 0\ \forall (x,t) \in \mathbf{U}.$$

A PDAE can represent a set of interrelated partial differential, ordinary differential, and algebraic (finite) equations, connected at least through some components of the unknown vector-functions, where x or t act as parameters.

In this talk, we discuss the concept of index for PDAEs, which characterizes the complexity of its internal structure based on the theory of $|l|$-prolonged systems [3], i.e. the collection of the original system (1) and its derivatives up to order $|l|$:

$$\Big\{\Lambda_k(D_t, D_x)u - f(x,t) = 0,\ \cdots,\ \frac{\partial^{i+j}}{\partial x^i \partial t^j}\Big[\Lambda_k(D_t, D_x)u - f(x,t)\Big] = 0,\ i+j \le l\Big\}.$$

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Winkler foundation based viscothermoelastic micro-schale circular plate resonators under moore-gibson-thompson theory

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The Winkler basis viscothermoelastic model combines the principles of the Winkler foundation with Kelvin-Voigt type viscothermoelastic structures. The appropriate implementation of the boundary conditions is the most rigorous part of the asymptotic techniques. Analytic as well as numerical results are produced for TED of circular micro-plate supposed to be based on Winkler foundation. The considered circular plate is Kelvin-Voigt type Moore-Gibson-Thompson viscothermoelastic (MGTTE) homogenous isotropic, thermally conducting supposed to be based on a Winkler foundation. The numerical illustrations have been accomplished [1] with help of MATLAB programming. The acquired findings are significant due to enhance precision and decrease in energy dissipation in the creation of such circular resonators. The effectiveness and accuracy have vital role in engineering applications like ramifications for contemporary design especially in dynamic loading conditions for various structures of high quality like beam, plate and slabs resting on Winkler foundation.

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A mixed FEM for PIDE using biorthogonal basis

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This study develops a mixed finite element approach based on a saddle-point formulation for solving parabolic biharmonic integro-differential equations of Kirchhoff type subject to simply supported boundary conditions. The well-posedness of the proposed formulation is rigorously established, and stability along with optimal error estimates are derived for both semi-discrete and fully discrete finite element schemes. To efficiently handle the memory term, a rectangle quadrature rule is employed for its numerical approximation. Several numerical experiments are presented to confirm the theoretical results and to demonstrate the robustness and accuracy of the method, including its performance on non-convex computational domains.

Lie Symmetry and Variational Analysis of a Blood Flow Model with Body Force

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This work presents a comprehensive analysis of a one-dimensional nonlinear blood flow model that incorporates a body force term, using both Eulerian and Lagrangian descriptions. By introducing Lagrangian coordinates, the system is reformulated as a single second-order partial differential equation derived from a variational principle. Lie symmetry analysis is performed in both coordinate systems, leading to the construction of one-dimensional optimal systems and exact invariant solutions. Variational symmetries satisfying Noether's criterion are identified, and the associated conservation laws are obtained using Noether's theorem. Finally, the evolution of weak discontinuity waves is investigated using an exact solution which reveals significant nonlinear effects such as wave steepening and shock formation. The results highlight the role of symmetries and conservation laws in understanding wave behavior in physiological flow models.

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On models of a slice-by-slice solution to a refractive tomography problem in the half-space

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The phenomenon of refraction of the ray along which the signal propagates occurs in the process of probing an inhomogeneous medium by a physical field of almost any nature. Within the framework of the X-ray tomography, this effect is reasonably neglected. But in a number of other formulations, for example, in seismics, refraction is significant. Moreover, it is the most important element of the model, which is necessarily includes it. We consider a version of the formulation of the refractive tomography problem, in which the refraction in the domain is modeled by a given Riemannian metric. In two-dimensional case the similar well known problem is solved by least squares numerical method using polynomials or B-splines as bases [1], [2]. The ray transform image of a 3D function depends on four variables, so the inverse problem becomes overdefined, so often In computerized tomography the 3D problem is reduced often to a family of 2D problems, each of which gives a trace of the desired 3D function on one of the families of parallel planes.

The initiation of the refraction phenomenon into the tomography model significantly complicates the problem, since an arbitrary Riemannian metric, as a rule, does not have families of completely geodesic 2D submanifolds similar to families of planes in Euclidean 3D space. The question arises about the existence of Riemannian metrics that allow one to reduce the 3D problem of refractive tomography to a series of two-dimensional ones, and, thus, to the standard method of slice-by-slice reconstruction of the desired function. Let in the half-space $\mathbb{R}^3_{x^3+} = \{x \in \mathbb{R}^3 | x^3 > 0\} \equiv D$ the isothermal Riemannian metric

$$ds^2 = \lambda^2(x, y, z)(dx^2 + dy^2 + dz^2)$$

with $\lambda(x, y, z) = (az + b)^{-1})$, $a, b > 0$, be given. A model of refractive tomography in a half-space is considered, allowing the application of the method of slice- by-slice solution of the 3D problem. It is known, that the system of equations of geodesics of such metric, is solvable in quadratures, and its geodesics are arcs of circles lying in planes parallel to the Z axis [3].

A refractive tomography problem consists in reconstruction of a function $\lambda(x, y, z)$ from its geodesic ray transform

$$\mathcal{P}(p, q) = \int_0^{t_+} \lambda(\gamma_{p,q}(t))dt, \quad p \in D \cup \partial D, \quad q \in \partial D,$$

where $\gamma_{p,q}$ is a geodesic line connecting points p, q. We establish families of completely geodesic 2D submanifolds in the half-space. An approach of slilce-by-slice solution for the cobsidered refractive tomography problem is proposed and substantiated by reducing the topological dimension to solving a family of two-dimensional problems. A 2D metric in a circle is constructed, isometric to the corresponding totally geodesic submanifold of the original metric in the half-space. Geometric characteristics of both the original isothermal metric and the 2D metrics generated by it are established. The approach allows reducing the solution of the 3D problem to the sequential solution of a series of 2D problems of refractive tomography.

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Using of conservation laws to construct an elastic-plastic boundary of twisted multilayer rods of a rolled profile reinforced with elastic fibers

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N. N. Yanenko was one of the first to use conservation laws to solve problems in mechanics [1]. To solve and study the equations of mechanics of a deformable solid, conservation laws were found and used by Senashov S.I. [2]. Currently, the main boundary value problems for the equations of plasticity, elastic plasticity and composite materials have been solved using conservation laws [3].

The paper studies the elastoplastic torsion of a multilayer rod of a rolled profile under the action of a torque. It is assumed that the rod is reinforced with elastic fibers. The layer contact boundary is located along the ox axis. The lateral boundary of the rod is stress-free, but the boundary is in a plastic state. The components of the stress tensor at a point are calculated using contour integrals obtained from conservation laws calculated from the lateral boundary and fiber boundary. Next, the second invariant of the stress tensor is compared with the yield strength. In those points where the yield point is reached, the plastic state is realized, in the rest it is elastic. This makes it possible to draw a boundary between the plastic and elastic regions. This technique provides a way to calculate the elastic-plastic boundaries for the main rolled rod profiles.

For this problem, an infinite system of conservation laws is constructed when the conserved current is linear relative to the components of the strain tensor. The use of singular solutions of the elasticity equations makes it possible to reduce the problem to calculating contour integrals along contour boundaries, layer boundaries, and fiber boundaries. A program written in the Maple environment is used to calculate integrals. It allows you to draw boundaries between elastic and plastic areas, which makes it possible to assess the strength of the rod under study.

The author expresses his gratitude to Professor Senashov S. I. for setting the task and paying attention to the work.

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A quantum time operator for parametrization invariant Lagrangians

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We start from a geodesic, parametrization-invariant Lagrangian, whose non-local symmetries allow us to identity the particle's proper time as a phase-space function. In the quantum description of the system, this quantity is promoted to an operator. Due to the symmetries of the conformal Laplacian, and the vanishing of the Hamiltonian, we demonstrate that this operator satisfies the defining properties of a time operator. We examine the possibility of the existence of a time-energy uncertainty relation and discuss the applicability of this construction to cosmological systems.

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Adaptive spline technique to solve singularly perturbed delay differential equation with mixed large shifts

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In this work, we solved singularly perturbed differential equations with mixed shifts. To reduce solution oscillation, we used the adaptive spline approach and applied a fitting factor. Numerical experiments are carried out based on the theoretical study to support the accuracy of estimations and validate the validity of our theoretical conclusions. In the maximal norm, the approach exhibits almost first-order convergence. We tabulated the maximum absolute errors and displayed the pointwise absolute errors using graphical representations. There were additional tables showing the rate of convergence as determined mathematically.

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Asymptotic solutions of multidimensional linear (pseudo)differential equations with time-harmonic and spatially-localized sources.

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We discuss the problem about short wave asymptotic solutions of linear multidimensional differential and pseudo-differential equations with a small parameter describing waves in inhomogeneous media generated by time-harmonic and spatially-localized sources. The problem is similar to the problem of the asymptotics of the Green function for stationary equations, but instead of a delta function side, there is a localized function on the right-hand. The simplest example of such a problem is the multidimensional Helmholtz equation with a variable potential and a localized right-hand side. Effective asymptotic formulas for solutions are constructed in the form of the Maslov canonical operator on Lagrangian manifolds, woven from suitable trajectories of a Hamiltonian system with the Hamiltonian equal to the principal symbol of the (pseudo)differential operator that defines the original equation. The formulas for the asymptotic solution preserve information about the shape of the source. We show that for some equations, in particular, the Helmholtz equation and the wave equation, the trajectories that define the specified Lagrangian manifolds can be found numerically using the Fermat variational principle.

The authors gratefully acknowledge financial support by the State Assignment 124012500442-3 (Russia).

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Invariant reduction of Poisson brackets

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For a system of differential equations $F = 0$ admitting a local (point, contact, or higher) symmetry X with a characteristic φ, invariant solutions satisfy the reduced system

$$F = 0\,, \qquad \varphi = 0\,.$$

There is a general framework [1] describing how the reduced system inherits X-invariant geometric structures from the original one. In particular, the reduction applies to conservation laws and presymplectic structures.

In this talk, we will extend this approach to the case of X-invariant Poisson brackets. Their reduction is based on the interpretation of local Hamiltonian operators as degree-2 conservation laws and degree-1 symmetries of degree-shifted cotangent equations.

In the setting of $(1+1)$-dimensional Hamiltonian systems, the reduced Poisson brackets define Poisson bivectors relating constants of X-invariant motion to symmetries of the reduced systems. In terms of the reduction of X-invariant conservation laws, these bivectors agree with the corresponding Hamiltonian operators of the original systems, up to sign.

In the case of $(1+1)$-dimensional evolution systems with $F^i = u^i_t - f^i$, this approach leads to the following result. Suppose that $\varphi = (\varphi^1, \ldots, \varphi^m)$ is the characteristic of a symmetry X such that its components φ^j do not depend on the variables u^i_t or their derivatives. Let $u^1_{k_1+1}$, …, $u^m_{k_m+1}$ denote the highest order x-derivatives among the arguments of φ.

Theorem 1. Suppose that the system $\varphi = 0$ can be solved in the variables $u^1_{k_1+1}$, …, $u^m_{k_m+1}$. If ∇ is an X-invariant Hamiltonian operator of the system $F = 0$ acting on vector functions $\psi = (\psi_1, \ldots, \psi_m)$,

$$\nabla(\psi)^i = \nabla^{ij\,k} D_{kx}(\psi_j)\,,$$

then on the algebra of constants of X-invariant motion, there exists a Poisson bracket inherited from the one defined by ∇. In coordinates of the form t, x, u^i_0, …, $u^i_{k_i}$, the bivector defining this bracket is given by

$$-\frac{1}{2}\Big(\widetilde{\nabla}^{ij\,k} w_{j\,k} \wedge \partial_{u^i_0} + \mathcal{L}_{\widetilde{D}_x}\big(\widetilde{\nabla}^{ij\,k} w_{j\,k}\big) \wedge \partial_{u^i_1} + \ldots + \mathcal{L}^{\,k_i}_{\widetilde{D}_x}\big(\widetilde{\nabla}^{ij\,k} w_{j\,k}\big) \wedge \partial_{u^i_{k_i}}\Big),$$

where $\widetilde{\nabla}^{ij\,k}$ and $\widetilde{D}_x$ denote the restrictions of the components and the total derivative to the reduced system, $\mathcal{L}_{\widetilde{D}_x}$ is the corresponding Lie derivative, $w_{j\,0}$ are the vector fields on the reduced system defined by the relations

$$\partial_{u^i_{k_i}} = -w_{j\,0}\, \frac{\partial \varphi^j}{\partial u^i_{k_i+1}}\bigg|_{\text{reduced system}}$$

and $w_{j\,k} = \mathcal{L}^{\,k}_{\widetilde{D}_x}(w_{j\,0})$.

The author gratefully acknowledges Alexander Verbovetsky for suggesting the idea of reducing Poisson brackets by interpreting them as conservation laws of the cotangent equations. He also thanks Eric Boulter and Raphaël Belliard for valuable discussions, the University of Saskatchewan for hospitality, Prof. Alexey Shevyakov for financial support through the NSERC grant RGPIN 04308-2024, and the Pacific Institute for the Mathematical Sciences for support through a PIMS Postdoctoral Fellowship.

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The Riemann spaces related to the Navier-Stokes equations

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The properties of the Navier-Stokes system of equations are studied on the basis of the associated 14-dimensional Riemannian space D14 with a zero Ricci curvature tensor is equal to zero on the solutions of the system. The geometric characteristics of the space under consideration are determined by the relationships between the components of the Riemann curvature tensor, depending on the functions of velocity and pressure of the fluid, which are then used to construct examples of exact solutions for both laminar and turbulent flows. By studying in detail the properties of geodetic lines of the D14-space metric, the conditions for the compatibility of a system of NS equations are studied, which lead us to the systems of complex linear equations of the Schrodinger type. As an example, we consider the six and four –dimensional Riemannian metrics with a zero Ricci curvature tensor on solutions of the Kadomtsev-Petviashvili equation which integrated by the method of the inverse problem from soliton theory.

Invariant Conservative Finite-Difference Schemes for Some Two-Dimensional Hydrodynamic-Type Equations

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We discuss methods and techniques for constructing invariant and conservative finite-difference schemes for two-dimensional hydrodynamic-type equations. Approaches to the construction of invariant (symmetry-preserving) finite-difference schemes have been actively developed since the late 1980s, notably in the works of Professor V.,A. Dorodnitsyn and his collaborators (many of the main results are presented in [6]). The symmetry properties of differential equations are closely related to the presence of conservation laws; consequently, invariant schemes are typically conservative.

Over the past decade, numerous invariant finite-difference schemes have been constructed for one-dimensional equations, including equations with various spatial geometries, primarily in Lagrangian coordinates [5, 4, 3]. In contrast, these methods have only rarely been applied to equations in higher spatial dimensions. Recently, standard approaches have been extended to the two-dimensional case for the shallow water equations in Lagrangian coordinates [1] and for two-dimensional gas dynamics equations describing dissociating gases [2].

It was shown [1] that, at the discrete case, certain restrictions arise on the forms of admitted symmetries when uniform orthogonal meshes are employed. In particular, the relabeling generator, which involves an arbitrary function and is typically admitted for hydrodynamic-type equations in Lagrangian coordinates, can no longer depend on a completely arbitrary function in the two-dimensional case if uniform orthogonal meshes are used.

We also briefly discuss the group foliation method for hydrodynamic-type equations in the context of the relabeling generator. This method represents the original system as two equivalent subsystems that are, in some cases, simpler than the original system. For hydrodynamic-type equations with a relabeling symmetry, these subsystems possess some notable properties.

The research was partially supported by the Russian Science Foundation, Grant No. 18-11-00238, “Hydrodynamics-type equations: symmetries, conservation laws, invariant difference schemes,” and by the Russian Science Foundation (Project No. 23-11-00027).

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How to determine the speed and amplitude of the leading edge of a dispersive shock wave

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The objective of my talk is to describe the solitary wave of largest amplitude in the dispersive shock appearing in the solution of Riemann problem for dispersive equations describing non-linear long dispersive waves, in particular, the Benjamin-Bona-Mahony equation and Serre-Green-Naghdi equations. Such a large-amplitude solitary wave is the leading wave of the corresponding dispersive shock. Its speed and amplitude are defined analytically through the solitary limit of the corresponding Whitham modulation equations. In such a limit, Whitham's equations form a system of quasi-linear equations for which Riemann's invariants can be determined. The numerical results are in accordance with the analytical prediction.

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Compact finite-difference scheme and modified Richardson extrapolation for the NLSE

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A compact finite-difference scheme combined with predictor-corrector approach for solving quasilinear partial differential equations and systems is presented. The nonlinear Schrodinger equation (NLSE) serves as a model problem to demonstrate the method's capabilities. The proposed algorithm achieves fourth-order spatial accuracy and second-order temporal accuracy while maintaining computational efficiency through linearization via Newton - Raphson iterations. Also we introduce a modified two-dimensional and quasi-two-dimensional Richardson extrapolation technique that further enhances accuracy up to eighth-order. Numerical experiments confirm the scheme's high precision and stability across a range of Courant parameters as well as a good conservation of many first integrals of NLSE. The method is applicable to arbitrary smooth initial data and various boundary conditions. We checked the properties for various solutions (solitons, collision of many solitons, modulation instability).

Analysis of Parkinson Disease Using Fuzzy C-Means Clustering Techniques

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This work presents an interpretable machine learning framework for diagnosing Parkinson's Disease using simplified models. Logistic Regression, Decision Tree, K-Nearest Neighbours, and Naive Bayes were evaluated on publicly available Parkinson's datasets. The results show that Decision Tree and Logistic Regression achieved the best performance, with diagnostic accuracy reaching approximately 85–90 percentage. Principal Component Analysis improved efficiency by reducing feature dimensionality without affecting accuracy. Fuzzy C-Means clustering revealed overlapping patient groups, reflecting early and borderline Parkinson's cases[1]. Overall, the study demonstrates that simple, explainable models which can deliver reliable, clinically meaningful diagnoses of Parkinson's disease[2].

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Thermochemical nonequilibrium as a factor of stabilization of supersonic boundary layers

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In the present time it has now been established that thermochemical nonequilibrium (TNE) can significantly affect the stability and laminar-turbulent transition in the boundary layer (BL). Research on the possibilities of using TNE was initiated by the experiments of Professor H. Hornung's group [1], which studied the effect on the laminar-turbulent transition (LTT) on the cone of an additive and the injection of a vibrational excited dissociating carbon dioxide into a hypersonic flow. The specific experimental conditions of H. Hornung's T5 Caltex setup, in which the carrier gas significantly dissociated due to high temperatures, make it impossible to isolate the contribution of the additive itself or the injection to the downstream LTT shift. The high level of flow disturbances characteristic of the T5 setup also complicates an objective assessment of the stabilizing role of CO2. In this regard, it is of interest to examine the influence of the additive and injection of carbon dioxide into the flow of non-dissociating nitrogen within the framework of a simple LTT problem in a supersonic BL on a plate. The choice of the N2/CO2 pair is due to the fact that the dissociation temperature of nitrogen is significantly higher than that of carbon dioxide, and the kinetics of CO2 's vibrational modes and dissociation-recombination reactions are the most studied among polyatomic gases. In addition, this allows for a qualitative comparison of the results with those of H. Hornung. Using a system of gas dynamics equations for a mixture of molecular vibrationally excited chemically reacting gases, the effect of a carbon dioxide additive on the stability of a supersonic BL in a mixture with neutral nitrogen on a plate was investigated. Calculations were performed for five mixture composition variants. The dependence of the LTT Reynolds number on the molar concentration of CO2 was calculated using the semiempirical eN - method. The resulting dependence is linear and coincides with the experimental dependence [1] in matches up to a parallel transfer. The effect of distributed carbon dioxide injection on the stability of a supersonic neutral nitrogen flow on a plate was studied. Calculations were performed for a number of fixed injection parameter (intensity) values. To assess the effect of injection intensity on the onset of the LTT zone, families of N-factor curves were calculated using the semiempirical eN-method. It was shown that maximum stability is achieved at high values of the injection parameter. This means that, under certain combinations of flow parameters and injected gas, injection not only effectively protects the surface but simultaneously improves flow stability.

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Exploration of Stability for Stationary Solutions to the Vlasov-Poisson Equations of Electron Plasma in One-Dimensional Setting

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The Vlasov-Poisson model of boundless collisionless electron gas in self-consistent electric field continues to be one of the basic models for a number of modern physics areas, such as particle physics, electrodynamics, plasma physics, etc. This is due to simplicity, clarity, and obvious effectiveness of the model in describing complicated processes of the micro world. For example, the Vlasov-Poisson model is successfully used for development and operation of accelerators with colliding beams, which make it possible to accelerate elementary particles additionally by means of hot electron gas.

In this report, the problem of linear stability for stationary solutions to the Vlasov-Poisson equations of electron plasma in one-dimensional formulation is investigated [1]. The research objective is to prove an absolute linear instability for these solutions with respect to small one-dimensional perturbations.

To achieve such goal, a transition from the Vlasov-Poisson equations to an infinite system of relations similar to the equations of isentropic flow of a compressible fluid medium in the "vortex shallow water" and the Boussinesq approximations was carried out. In the course of instability proof for stationary solutions to the Vlasov-Poisson equations in both kinetic and gas-dynamic descriptions, the well-known sufficient Newcomb-Gardner-Rosenbluth condition for stability of these solutions with regard to some incomplete unclosed subclasses of small one-dimensional perturbations was conversed. Also, the Gubarev inequalities were obtained for the Lyapunov functionals. The a priori exponential lower estimates for growth of small one-dimensional perturbations follow from these inequalities when the sufficient conditions for linear practical instability of the considered stationary solutions found in this report are satisfied. Since the obtained estimates were deduced without any additional restrictions on stationary solutions under study, then an absolute linear instability for these solutions with respect to small one-dimensional perturbations was thereby proved.

The report results are confirmed by analytical examples of stationary solutions to the Vlasov-Poisson equations. Besides, they are fully consistent with the classical Earnshaw instability theorem and expand the area of applicability for this theorem from electrostatics to kinetics.

Finally, constructiveness is inherent in the sufficient conditions for linear practical instability established here, which allows them to be used as a testing and control mechanism for conducting physical experiments and performing numerical calculations.

In particular, these conditions can be applied to solve the problem of additional acceleration for elementary particles by electron plasma clumps. Specifically, they must be performed during the formation of a bunch of electrons. Due to this, it will take less time to create an electronic clot. On the contrary, after the electron cluster has formed, the conditions should not be fulfilled. Then the electronic clot will last longer. Thus, there is a real opportunity to control the acceleration of elementary particles with the help of electron plasma clumps.

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On the construction of solutions of the Davey-Stewartson I equation via dressing chain

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An original effective method for constructing explicit solutions of integrable Davey-Stewartson type equations is proposed, based on the use of dressing chains. The main difficulty arising when using the symmetry approach in 3D is associated with non-local variables entering the equation. To solve the nonlocality problem, it is proposed to replace the infinite dressing chain with its finite field reductions preserving the integrability property. The application of the method is illustrated by the example of the DS I equation, for which a new class of explicit solutions is constructed that depend on two arbitrary functions. In this example, the dressing chain is replaced by a finite field reduction of the Toda lattice corresponding to a simple Lie algebra A_2.

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A pest management SI epidemic model with instantaneous and non-instantaneous impulsive effects

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We propose and analyze a Susceptible-Infected (SI) epidemic model applied to pest management, focusing on the nonlinear release of infected pests and an instantaneous pulse of pesticide spraying. Additionally, the mortality rates of both susceptible and infected pests following the pesticide application are modeled as non-instantaneous pulses. Utilizing the comparison theorem for pulse differential equations and Floquet theory, we derive a threshold condition for the eradication of susceptible pests. We also demonstrate that all solutions are uniformly ultimately bounded. Furthermore, we establish conditions for the globally asymptotic stability of the pest-free boundary periodic solution and the permanence of the system. Finally, numerical simulations are conducted to verify the theoretical findings, and the key parameters affecting the pest extinction threshold are obtained, thereby providing a solid theoretical basis for the development of effective pest management strategies.

Stability of a compressible gas layer in a gravity field

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The equilibrium stability of a compressible gas layer in a gravitational field is numerically investigated. A uniform temperature is maintained at all boundaries of the computational domain. Using a linear approximation, it is shown that at a sufficiently high altitude, the static equilibrium solution becomes unstable. This instability is due to the pressure change with altitude due to gravity and the compressibility of the medium. The obtained data are confirmed and supplemented by the results of solving a system of complete nonlinear equations describing the flow of compressible gas. The features of the resulting non-stationary solution are discussed [1]. The problem of studying the stability of a compressible gas layer in a gravitational field in the absence of specified temperature gradients is of particular practical and theoretical interest due to its direct connection with the issue of explosion safety of fuel storage in tanks. Indeed, an explosive situation arises when a tank containing previously stored fuel is almost empty and the remaining fuel at the bottom evaporates, forming a fuel-air (steam-air) mixture. However, as the results of the linear analysis show, the static equilibrium state of such a vapor-air gas mixture becomes unstable at a sufficiently high gas layer height due to pressure changes with altitude and the compressibility of the medium. The resulting flows ensure effective mixing of the fuel-air mixture, potentially creating regions with explosive fuel concentrations. The explosion hazard of vapor-air gas mixtures has been discussed in many works; references can be found in the monograph [2].

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Interpretation of Actuarial Science Models Using Mathematical Techniques

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The given research is dedicated to the possibilities of using the advanced mathematical and calculative methods in order to improve the prediction and decision-making in Finance datasets and in actuarial practice[1, 2].The work highlights the relevance of the data-driven models in the successful management of large and complex data. The modern methods of analysis are discussed in the context of its capability to deal with ambiguities and enhance accuracy. The results show that these methods are even more efficient and reliable in comparison with traditional methods. There is less estimation error and greater consistency that is seen in various applications. The paper outlines the possibilities of highly detailed modeling techniques to be used in the research in the future.

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Operational formulas of Appell-type telephone-Hermite polynomials

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In this work, we present operational formulas for an Appell-type family of polynomials, called the telephone polynomials, which generalize the classical telephone (involution) numbers, denoted by $T_n(x)$[1] and defined as

$$T_n(x) = xT_{n-1}(x) + (n-1)T_{n-2}(x) \quad \text{with} \quad T_0(x) = 1, T_1(x) = x$$

and its closed form expression is given by

$$T_n(x) = \sum_{r=0}^{[n/2]} \frac{n!}{2^r(n-2r)!r!} x^{n-2r}.$$

We establish various telephone-Hermite connections, recurrence relations, differential equation, etc. These operational results reveal direct connections with classical Hermite, Laguerre, and Legendre polynomials, leading to new identities among these families[2]. The Appell-type nature of the telephone–Hermite polynomials makes them suitable for unified operational and computational treatments of special polynomial systems.

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A Fractional Step Numerical Framework for Fuzzy Preference Dynamics in Music Recommendation Systems

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A numerical framework is presented for modeling and simulating preference evolution in music recommendation systems using fuzzy differential equations coupled with data-driven learning mechanisms. User preferences and perceptual attributes of music are represented as fuzzy states evolving over time, capturing the intrinsic uncertainty and subjectivity inherent in listening behavior. The resulting fuzzy dynamical system is realized numerically through a method of fractional steps, in which fuzzy inference, learning-based operator approximation, and preference evolution are treated as distinct subproblems. Machine learning models are employed to approximate nonlinear fuzzy operators and interaction terms that are difficult to characterize analytically, while classical numerical schemes are used to advance the underlying differential equations. This operator-splitting strategy enables a stable and interpretable integration of fuzzy logic with learning components, consistent with Yanenko's methodology for complex dynamical systems. Numerical experiments on music preference data demonstrate the effectiveness of the proposed approach in capturing temporal adaptation, uncertainty propagation, and convergence properties of the recommendation dynamics. The proposed framework illustrates how hybrid fuzzy–learning systems can be systematically analyzed and implemented within a rigorous numerical modeling paradigm, with music recommendation serving as a representative application of broader relevance to data-driven dynamical systems under uncertainty.

keywords: Fuzzy differential equations, Operator splitting methods, Numerical modeling, Data-driven dynamical systems, Music recommendation systems.

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Solutions to the Riemann–Euler equations and their applications

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This report considers one-dimensional equations of acoustics equations of inhomogeneous media and the system of gas dynamics equations with constant entropy. Using the Riemann approach, the gas dynamics equations are reduced to a second-order linear hyperbolic equation with variable coefficients. Solutions to this equation are constructed using Euler–Darboux transformations. This allows us to find new exact solutions of the equations of acoustics and gas dynamics, depending on two arbitrary functions.

Improving image denoising via a nonlocal anisotropic diffusion model based on Caputo fractional derivatives and Gaussian convolution

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In the field of image restoration, denoising is considered one of the most important techniques. It is a preprocessing approach aims to refine image clarity and enhance its overall quality by effectively reducing noise present in the image. The aim is to obtain good-quality images from a version degraded by additive noise or convolutional noise that introduces blur. As a result, more advanced treatments can be performed on the resulting image. In order to remove Gaussian noise from input images, we propose the following methodology using a fractional differential equation in time-space based on Gaussian convolution, where the integer and fractional order derivatives of Caputo can be discretized using finite difference and L_1-approximations. Once the equation is solved numerically, the scheme is applied to grayscale digital images using the presented algorithm. The parameters must be optimized and adjusted. As a result of testing with natural images, we are able to successfully suppress the noise present in the images. Aside from that. Our model demonstrates strong visual quality, as verified by the calculation of indexes such as Peak Signal-To-Noise Ratio (PSNR) and Structural Similarity Index Measure (SSIM). Our denoising technique demonstrates its effectiveness in mitigating noise present in both MRI and X-ray image

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Symmetries of Multi-Fluid Scalar Field Cosmologies

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This work reviews the application of symmetries of simplest and highest order in the context of cosmological models with specific application towards dark matter and perfect fluids[1, 2]. The symmetries leads to obtaining new conserved densities which in turn leads to analytical solutions. Also, we highlight certain Machine learning techniques which could also provide a new perspective to the study of these models.

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Tempered Ψ-Hilfer Fractional Derivative and Fractional Differential Equations Involving It

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In this talk, we compare tempered and untempered fractional kernels, highlighting how tempering modifies the classical power-law behavior and leads to more realistic for systems with fading memory effects. We introduce a definition of the tempered Hilfer fractional derivative operator with respect to nonearning function Ψ- (called tempered Ψ–Hilfer fractional derivative operator) and investigate some of its fundamental properties. The proposed operator is sufficiently general and unifies a wide class of existing fractional derivatives, including tempered fractional derivatives as well as classical fractional derivatives. We further discus the mapping and limit properties of the tempered Ψ–fractional integral and develop a framework of Ψ–tempered fractional calculus, which is essential for deriving equivalent formulations of fractional integral and fractional differential equations involving the tempered Ψ–Hilfer fractional derivative. Finally, we discuss existence, uniqueness, and stability results for fractional differential equations governed by this operator.

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Improving the Accuracy of Shock Capturing Schemes Using the Richardson Extrapolation

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We perform a comparative analysis of accuracy of the CU (Central Upwind) [1] and A-WENO (Alternative Weighted Essentially Non-Oscillatory) [2] schemes with the QL (Quasi-Linear) NFD (New Finite-Difference) scheme [3] when solving two SCP (Special Cauchy Problem) problems [4] with discontinuous periodic initial data for shallow water equations. In solving the SCP1 problem, the accuracy of all three schemes in the shock influence areas and within centered rarefaction waves is comparable. In solving SCP2 problem, which exact solution contains shocks but does not contain rarefaction waves, the NFD scheme has several orders of magnitude higher accuracy in the shock influence areas than the CU and A-WENO schemes. To increase the accuracy of combined schemes where the NFD scheme is the basic and the CU or A-WENO schemes are the internal ones, we apply the Richardson extrapolation to the numerical solution obtained by the NFD scheme in those regions of the solution where there are no noticeable nonphysical oscillations. In calculating both SCP problems, the new combined schemes [5, 6] provide significantly higher accuracy (for example, see Table 1) compared to the CU and A-WENO schemes, both in the shock influence areas and within centered rarefaction waves.

x	0.4	1.4	2.4	3.4	4.4	5.4	6.4	7.4	8.4	9.4
CU	−4.24	−4.26	−4.33	−4.45	−3.40	−4.11	−4.16	−4.25	−4.32	−4.42
A-WENO	−4.54	−4.24	−4.48	−4.59	−3.81	−4.14	−4.15	−4.27	−4.40	−4.43
NFD	−5.59	−7.42	−7.40	−7.87	−3.83	−7.00	−7.24	−7.26	−7.22	−8.56
NFDR	−7.71	−8.14	−8.38	−9.49	−5.37	−7.38	−7.83	−7.84	−8.00	−9.89

Table 1: The averaged local disbalances obtained at the time moment $T = 1.5$ when calculating the SCP2 problem by the CU, A-WENO, NFD and NFDR (NFD with applying the Richardson extrapolation) schemes.

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Semi-smooth Newton Method for Contact and Dynamic Problems

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Motivated by inequality constraints appearing in convex and non-convex optimization context, we study non-smooth problems with respect to its properties of generalized differentiability. The use of Lagrange multipliers and merit functions leads to a semi-smooth Newton method for solution of linear and nonlinear complementarity problems and equivalent primal-dual active-set numerical algorithms. For application in mechanics, problems describing cohesive obstacle, two-body contact, non-penetrating cracks and fluid-driven fractures are considered. In the semi-smooth framework, we investigate contact and impact dynamics, in particular, the dynamic contact problem between beam and foundation under moving load stemming from railway applications.

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The Kuramoto-Sivashinsky equation with nonlinear convection: reduction, Painlevé test, first integrals and general solution

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This report is devoted to the study a generalization of the Kuramoto-Sivashinsky equation in the form [1]

$$u_t + u_{xx} + \sigma\, u_{xxx} + u_{xxxx} + \beta\, u\, u_x + \chi\, (u^2)_{xx} = 0.$$

This equation is used to model processes in combustion physics, plasma physics, hydrodynamics, and other fields. Since the Cauchy problem for this equation cannot be solved by the inverse scattering transform, a traveling wave reduction is employed to seek solutions. It is shown that the equation generally fails the Painlevé test. However, constraints on the model parameters are identified under which the corresponding nonlinear ordinary differential equation (ODE) [2]

$$C_1 - C_0\, y + y_z + \sigma\, y_{zz} + y_{zzz} + \frac{1}{2}\beta\, y^2 + \chi\, (y^2)_z = 0,$$

where C_1 is an arbitrary constants, $u(x,t) = y(z)$ and $z = x - C_0\, t$. PDE and ODE does not pass the Painlevé test in the general case but we find parameter of equation values, the necessary condition for the existence of a general solution is satisfied. Using the results of the Painlevé analysis, first integrals for the nonlinear ODEs are derived. The general solutions for two specific cases, each involving four arbitrary constants, are presented in terms of the Weierstrass elliptic function and the transcendental solutions of the first Painlevé equation. Furthermore, exact solutions with one and two arbitrary constants are obtained using the simplest equation method.

This research was supported by the Ministry of Science and Higher Education of the Russian Federation according to the state task project No. FSWU-2026-0006.

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Iterative learning control for Hilfer-type fractional stochastic differential systems: a simulation study for robotic applications

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This paper investigates iterative learning control for stochastic differential systems of fractional order in the Hilfer sense. Unlike existing studies that treat either fractional dynamics or stochastic effects separately, we develop an integrated framework that combines Hilfer fractional derivatives, Brownian perturbations, and a proportional–fractional integral learning law. The proposed approach captures both the memory effects and random uncertainties inherent in complex systems. As a case study, we apply the method to a gantry robot equipped with a flexible arm. Numerical simulations show that the Hilfer derivative significantly improves tracking accuracy and convergence speed compared to integer-order models, highlighting the potential of the proposed strategy for robotic applications under uncertainty.

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Heat and Mass Transfer Analysis over a Multilayer Flow in Cooling of Electronic Devices with Layered Microchannels

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Efficient thermal management of high-power electronic devices is a critical challenge due to escalating heat fluxes and nanosized particles. In this study, a comprehensive heat and mass transfer analysis of multilayer fluid flow in layered microchannels is analysed to the cooling of electronic devices. The physical model consists of a three-layer channel configuration, distinct fluid layers with dissimilar thermo-physical properties interact through conjugate heat and mass transfer mechanisms. Multilayer arrangements are increasingly employed in modern microelectronic cooling architectures to enhance thermal uniformity, reduce hotspot formation, and improve device reliability. The governing nonlinear coupled momentum, energy, and concentration equations are formulated for each layer by incorporating microscale effects, interfacial continuity conditions. To obtain analytical solution for this system, a regular perturbation technique is employed by introducing a small dimensionless parameter associated with flow inertia or thermal diffusion. This approach yield closed-form approximate solutions that elucidate the influence of key parameters such as thermal conductivity ratios, diffusivity ratios, heat source strength, and interfacial resistance on velocity, temperature, and concentration distributions. To complement the analytical treatment and ensure numerical accuracy, the reduced boundary value problem is solved using the fourth-order Runge–Kutta (RK4) method combined with a suitable shooting technique. The numerical solutions validate the perturbation results and enable detailed examination of nonlinear effects. Good agreement between perturbation and RK4 solutions is observed within the convergence range, confirming the robustness of the proposed methodology. The results demonstrate that multilayer channel configurations significantly enhance heat removal capability. The findings provide valuable design guidelines for next-generation electronic cooling systems, including microprocessors, power electronics, MEMS devices, data centres, and compact heat exchangers.

Global Solutions to the Discrete Nonlinear Breakage Equations without Mass Transfer

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We study a class of discrete nonlinear breakage equations describing collision-induced fragmentation processes without mass transfer. In this framework, clusters are characterized by their discrete size and collisions may cause one of the interacting clusters to split into smaller fragments while the other remains unchanged, preserving the total mass of the system. Such models arise naturally in the description of fragmentation phenomena in physics and related applied sciences.

The mathematical analysis of coagulation and fragmentation equations has a long history. A comprehensive theory for coagulation equations and fragmentation models with linear (spontaneous) breakage is developed in the monograph [4]. In contrast, nonlinear or collision-induced breakage equations, where fragmentation occurs as a consequence of particle collisions, exhibit substantially different mathematical structures and analytical challenges.

Early studies of discrete collision-induced breakage equations focused on models allowing mass transfer during collisions, which may generate clusters larger than the incoming ones, see for instance [3]. Excluding mass transfer leads to a qualitatively different dynamics, as fragmentation cannot produce larger clusters and the system is expected to evolve toward smaller sizes.

The aim of this work is to establish global well-posedness results for the discrete nonlinear breakage equations without mass transfer under minimal assumptions on the collision kernel. In particular, no growth condition is imposed on the kinetic coefficients, which are only assumed to be non-negative and symmetric. Under suitable assumptions on the fragment distribution function ensuring conservation of mass, we prove the existence of global mass-conserving mild solutions.

Under additional structural assumptions on the kinetic coefficients and boundedness conditions on the fragment distribution function, we further show that the mild solutions are classical. In this setting, uniqueness is established for initial data possessing finite higher-order moments. The proof is based on weighted stability estimates adapted to the nonlinear structure of the breakage operator, extending the approach developed in [2].

Finally, we investigate the large-time behavior of solutions and show that the system converges toward a monodisperse equilibrium consisting solely of monomers. A detailed presentation of the results can be found in [1].

Theorem 1. *Under suitable assumptions on the collision kernel and the fragment distribution function, the discrete nonlinear breakage equation without mass transfer admits a global mass-conserving mild solution.*

M. A. gratefully acknowledges the support of Jindal Global Business School, O.P. Jindal Global University.

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An Analytical Study of Thermally Affected Flow in Curved Pipes

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A steady, incompressible flow of a Newtonian fluid through a curved pipe of circular cross-section is studied under thermal effects. The continuity, momentum, and energy equations are expressed in toroidal coordinates which are highly coupled non-linear partial differential equations. Analytical solutions are obtained using a regular perturbation expansion with respect to the small curvature ratio. The zeroth-order solution corresponds to Hagen–Poiseuille flow, while higher-order terms capture curvature-induced secondary motion and thermal coupling. The effects of pertaining parameters on axial velocity, streamlined, fluid temperature and volume flow rate are examined. It is shown that increasing Reynolds number, even for small curvature, significantly enhances secondary flow, producing behavior comparable to that in highly curved pipes. Thermal effects are also discussed through the Prandtl number and an imposed axial temperature-gradient parameter.

A Robust Kinetic Meshfree Method for Anisotropic Point Clouds

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The Least Squares Kinetic Upwind Method (LSKUM) [1] is a meshfree scheme that belongs to the class of kinetic theory based numerical schemes for compressible fluid flows. It operates on a distribution of points, known as a point cloud. The point cloud can be obtained from simple or chimera grid generation algorithms, quadtree methods, or advancing front methods. Each point in the cloud requires connectivity or neighbourhood information. The number of points in the connectivity can vary over the computational domain. The basic idea of LSKUM is to introduce upwinding into the governing Euler or Navier-Stokes equations through kinetic flux vector splitting. Later, the kinetic split flux derivatives are approximated using a weighted least-squares formulation with appropriate connectivity sets that satisfy the upwind property. Over the past two decades, LSKUM based meshfree solvers have been successfully applied to a wide range of fluid flow problems in steady and unsteady flows [2, 3, 4].

The robustness and accuracy of LSKUM depend on the goodness of the connectivity, which can be quantified by the condition number of the weighted least-squares matrix. In computational domains with highly stretched or anisotropic connectivity sets, these matrices often become ill-conditioned, leading to loss of accuracy, oscillatory convergence behavior, or even code divergence [5, 6]. Although various strategies have been proposed to overcome these challenges [5, 6], they are heuristic and do not aim to minimise condition numbers.

This paper presents the development of optimally weighted LSKUM that yields minimal condition numbers. The optimal distribution of weights is found using a gradient based optimisation algorithm combined with discrete adjoints. The robustness and accuracy of the optimally weighted LSKUM are assessed by applying it to standard test cases for inviscid flows on highly stretched and anisotropic point clouds. Numerical results show that the optimally weighted LSKUM yielded a more accurate solution with better convergence in the residue than the existing weighting strategies.

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Bifurcations in solitary wave dynamics for models close to modified Korteweg – de Vries equation

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The work examines the solitary wave behaviour in an equation close to the modified Korteweg – de Vries (mKdV) equation

$$u_t + \Delta u_x + 6u^2 u_x + u_{xxx} + \varepsilon R(u, x) = 0, \qquad \Delta \in \mathbb{R},\ \varepsilon << 1$$

The mKdV equation itself ($\varepsilon = 0$) is a classical integrable model and can arise in the description of nonlinear wave processes in plasma, fluids, and nonlinear optics [1, 2]. But what happens to a soliton in a non-integrable case ($\varepsilon \neq 0$), for example, when viscosity, external force or pumping (damping) are taken into account? From the energy balance equation, it follows that

$$\frac{d}{dt}\int_{-\infty}^{+\infty} \frac{u^2}{2} dx = -\varepsilon \int_{-\infty}^{+\infty} uR(u, x) dx$$

and after substituting a solitary wave with slowly varying parameters, we obtain a dynamical system describing its amplitude and phase. This work analyzes this system for specific types of $R(u, x)$. A bifurcation analysis of the system is presented, showing its dependence on the parameters characterizing viscosity and damping/pumping. The conditions for the capture of a solitary wave by an external force are investigated. The obtained asymptotic results are compared with direct numerical simulations. Asymptotic and numerical results are in agreement for all types of equilibrium points in the obtained dynamical system [3, 4].

The research is carried out with the support of the Russian Science Foundation Grant No. 24-47-02007.

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Nonlinear Wave Patterns in the Two-Dimensional Cubic Complex Ginzburg–Landau Equation

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In this work, the two-dimensional cubic complex Ginzburg–Landau equation is studied using two ansatz-based analytical methods. This equation emerges as a reduced model for protein–lipid interaction dynamics in living cells, obtained from a reaction–diffusion system describing membrane-associated proteins [1, 2]. Several classes of wave structures, including solitary and periodic solutions are obtained. Parameter constraints ensuring the existence of the solution families are explicitly determined. Graphical representations of the obtained solutions are presented to illustrate their spatiotemporal behavior and to highlight the effects of key parameters. The results provide analytical insight into diffusion-driven wave propagation and pattern formation relevant to protein–lipid interactions on biological membranes.

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Analysis of Epidemic Models through Singularities

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The epidemic model, M-SDI(Multiple–Information Susceptible–Discussing–Immune) and SUQC (Susceptible–Unquarantined–Quarantined–Confirmed), is studied using singularity analysis to analyse the movable singularities and henceforth to derive the series solution around them, which in turn would provide the necessary foreground detailing the impact of each compartment with regard to any disease. The model has also been analysed with respect to determining its bifurcation points and plotting the graphs, which would illustrate the chaotic nature of the epidemic model[1]. The results of the M-SDI and SUQC model were compared and it was observed that both models reveal critical thresholds and behavioural transitions that are essential for informed decision-making together they enhance our understanding of dynamic systems and support effective strategies in and public health contexts[2].

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On the Generalized Tzitzeica Curve Equation

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Introduced in the early 1900s, Tzitzeica curves and surfaces remain an active topic of study due to their geometric significance as the first examples of centro-affine invariants [**?**]. A smooth, regular skew curve is called a Tzitzeica curve if the ratio of its torsion τ to the square of the distance d from the origin to its osculating plane is constant [**?**]. This talk presents a generalization of the Tzitzeica curve equation based on a new centro-affine invariant. A Wronskian-involving [**?**] third-order nonlinear ordinary differential equation corresponding to the generalized Tzitzeica curves is derived and expressed in terms of the curve's defining functions [**?**]. Several intriguing solutions of this generalized equation are discussed in relation to the classical Tzitzeica curves.

Conservation laws for nonlinear Schrödinger equations of general form

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The report examines a family of generalized nonlinear Schrödinger equations [1]

$$\begin{gathered} iq_t + \left(F_1(|q|^2)q\right)_{xx} + \left(F_2(|q|^2)q\right)_{xt} + H(|q|^2)q + iG_1(|q|^2)q_x + iG_2(|q|^2)q^2q_x^* + \\ iL_1(q_{xxx}, q_{5,x}, ..., q_{2n+1,x}) + L_2(q_{4,x}, q_{6,x}, ..., q_{2m+2,x}) = 0, \end{gathered} \tag{1}$$

where $q(x,t)$ is a complex-valued function, x and t are independent variables, $i^2 = -1$, $F_1(|q|^2)$, $F_2(|q|^2)$, $H(|q|^2)$, $G_1(|q|^2)$ and $G_2(|q|^2)$ are arbitrary real functions, $L_1(q_{xxx}, q_{5,x}, ..., q_{2n+1,x})$ and $L_2(q_{4,x}, q_{6,x}, ..., q_{2m+2,x})$ are linear functions of their arguments.

$$\begin{aligned} L_1(q_{xxx}, q_{5,x}, ..., q_{2n+1,x}) &= \sum_{j=1}^{n} a_{2j+1}q_{2j+1,x}, \\ L_2(q_{4,x}, q_{6,x}, ..., q_{2m+2,x}) &= \sum_{k=1}^{m} b_{2k+2}q_{2k+2,x}, \end{aligned} \tag{2}$$

$n \in \mathbb{N}$, $m \in \mathbb{N}$, $a_{2j+1} \in \mathbb{R}$, $b_{2k+2} \in \mathbb{R}$ and $q_{l,x} = \frac{\partial^l q}{\partial x^l}$. Considered nonlinear partial differential equations depend on arbitrary functions, and the Cauchy problem for them cannot generally be solved by the inverse scattering transform. The functions presented in Eq. (1) have a physical meaning and are associated with processes occurring during the propagation of an optical pulse in a nonlinear medium. The function $F_1(|q|^2)$ corresponds to diffraction scattering during wave propagation, while $F_2(|q|^2)$ is related to an additional frequency shift. The function $H(|q|^2)$ characterizes the nonlinear dependence of the refractive index of a light pulse in an optical medium. The dependencies $G_1(|q|^2)$ and $G_2(|q|^2)$ make it possible to account for the effects of self-steepening of the pulse front and stimulated Raman scattering. The functions $L_1(q_{xxx}, q_{5,x}, ..., q_{2n+1,x})$ and $L_2(q_{4,x}, q_{6,x}, ..., q_{2m+2,x})$ are responsible for higher-order nonlinear effects, specifically characterizing odd-order and even-order dispersion.

We construct conservation laws for Eq. (1). To achieve this, a modified multiplier method based on direct transformations of the equation is used [2, 3, 4]. We demonstrate that under certain constraints on the functions, the equation under study possesses several conservation laws. The paper presents a series of theorems on the construction of conservation laws with the obtained densities and fluxes.

This research was supported by the Ministry of Science and Higher Education of the Russian Federation according to the state task project No. FSWU-2026-0006.

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Propagation of Nonlinear Waves in the Combined Kairat-II–X Equation

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This paper addresses a nonlinear evolution model formulated as a combined Kairat-II–X equation [1, 2] that models nonlinear wave propagation involving the interplay of nonlinearity, dispersion, and higher-order effects. Analytical solutions of the equation including solitary, kink-type, and periodic structures are obtained using auxiliary equation methods. Numerical plots are included to visualize the qualitative features of the solutions and to demonstrate the role of the parameters. Owing to its applications in fluid dynamics, plasma physics, nonlinear optics, and ocean wave dynamics, the results contribute to a deeper understanding of nonlinear wave phenomena governed by the combined Kairat-II–X equation.

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Slicing the PDE's systems

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In this lecture, we will show, using the example of the system of gas-dynamics and magneto-gas-dynamics equations, how a system can be split into smaller ones. The possibility of splitting arises when the nonlinear interactions of a certain group of modes do not involve other modes outside this group. Therefore, by studying possible splittings, we examine the nonlinear couplings present in the system. As one might expect, the method is closely related to the idea represented by Yanenko and his school, namely the method of differential constraints.

The Extensive Study of Rossby Type Waves in Magnetohydrodynamics

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Solitary waves are one of the natural wave phenomena with the most theoretical importance and research value in many fields such as fluid and solid mechanics, geophysics, etc. One of them is Rossby waves, also known as planetary waves, which occur naturally in rotating fluids under the influence of the Earth's motion [1, 2, 3]. These waves are obtained as solutions to nonlinear evolution equations, of which the Korteweg-de Vries (KdV) equation, which models free-surface gravity waves propagating in shallow water, is one of the best-known. The quasi-geostrophic potential eddy equation (QGPVE) is derived from a reasonable simplification of the shallow water equations and describes the laws governing large- and mesoscale motions and Rossby type waves (RW) can be obtained [4, 1, 2] . This study depends on the solutions of KdV-type equations for RW and RKEM waves, so the derivation of the models has been studied in the literature in detail. Explicit analytical solutions are important to see the effects of the parameters. This work is designed as an analytical study to find exact solutions of the considered equation via symbolic computation methods, and the results are proposed and supported by plots.

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Analytical and Numerical Approaches to Nonlinear Differential Equation Models

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Nonlinear differential equations play a central role in modelling complex phenomena arising in mathematical, physical, and biological systems. Understanding the qualitative behavior of such models often requires a careful combination of analytical techniques and numerical investigations. In this talk, I present a systematic study of nonlinear ordinary and partial differential equation models using analytical methods supported by numerical simulations.

The analysis begins with the formulation of mathematically well-posed models governed by nonlinear differential equations. Existence of equilibria and their stability properties are investigated using linearization techniques and qualitative theory of differential equations. Special emphasis is given to bifurcation analysis to explore how system dynamics change under parameter variations.

For spatially extended systems, reaction–diffusion equations are considered to study the influence of diffusion on system stability. Analytical conditions for diffusion-driven instability are discussed, providing insight into the emergence or suppression of spatial patterns. Numerical simulations are employed to illustrate analytical findings and to visualize temporal and spatial dynamics, thereby bridging theory and computation.

The talk demonstrates how analytical tools such as stability theory, bifurcation analysis, and qualitative methods, when combined with numerical techniques, offer a comprehensive framework for understanding nonlinear differential equation models. The presented approach is applicable to a broad class of problems in applied mathematics.

Analytical Methods and Exact Solutions of Fractional-order Partial Differential Equations

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The main aim of this work is to investigate how to compute the exact solutions using the invariant subspace method for the time-fractional nonlinear partial differential equations under the ψ-Hilfer fractional-order derivative. We also explicitly demonstrate the importance and utility of the invariant subspace method in computing exact solutions for time-fractional diffusion equations. More specifically, we demonstrate systematically how to compute the linear spaces associated with the above-mentioned equations using the invariant subspace approach. Furthermore, the computations of exact solutions are investigated for the linear and nonlinear diffusion equations under the above-mentioned time fractional-order derivative with the help of the computed invariant linear spaces. Additionally, we notice that the computed solutions of the considered equations under the ψ-Hilfer fractional derivative are valid under the ψ-Riemann-Liouville, ψ-Caputo, Hilfer, Katugampola, Caputo-Katugampola, Riemann-Liouville, and Caputo fractional derivatives because the ψ-Hilfer fractional derivative is a generalization of those fractional derivatives. Also, note that the computed exact solutions to the underlying equations under the discussed fractional-order derivative are expressed in terms of trigonometric, exponential, and polynomial functions with two or three parameters of Mittag-Leffler functions. Finally, the exact separable solutions are presented for the initial and boundary value problems (IBVPs) of the discussed equations under various fractional-order derivatives and their comparison.

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Sturm's theorems in terms of generalized derivatives

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We define the Sturm separation and Sturm comparison theorems for the generalized derivative. The generalized derivative is defined in terms of the weight function and another function. We also define the generalized Sturm–Liouville problem (GSLP) and analyze its properties. We discuss the spectral properties of the GSLP such as eigenvalues are real and the associated eigenfunctions are orthogonal. Moreover, we use a variational approach to show that GSLP has infinite eigenvalues in increasing order.

Intrinsic phenomena of delta shock waves in a more realistic Chaplygin Aw-Rascle model

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The motivation of this study is to find the Riemann solutions of Aw-Rascle model with a more realistic version of extended Chaplygin gas. Firstly, we establish the Riemann solutions with two different structures, viz., a shock wave followed by the contact discontinuity and a rarefaction wave followed by the contact discontinuity. Further, by analyzing the limiting behavior, it is found that one of the Riemann solutions converges to δ-shock solution as the pressure approaches to generalized Chaplygin gas pressure. Moreover, numerical simulations have been performed to validate the theoretical analysis.

Keywords: Riemann problem; Chaplygin gas; Delta shock; Transport equations; Contact discontinuity

Note: This paper has also been published in "Applicable Analysis (Taylor & Francis)".

Publication Detail: Priyanka and M. Zafar, *Intrinsic phenomena of delta shock waves in a more realistic Chaplygin Aw-Rascle model,* **Applicable Analysis (Taylor & Francis)** (2025), DOI: 10.1080/00036811.2025.2537681.

Axisymmetric Boundary Layers in Second Grade Fluid

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To describe the motion of relaxing fluids, such as aqueous polymer solutions, a second grade fluid model is used [1, 2]. For the first time, the Marangoni and Prandtl axisymmetric boundary layer equations were derived in a second grade fluid, and an initial-boundary value problem was formulated. The free boundary is assumed to be planar. This assumption is justified for small capillary numbers. Thermocapillary forces act on the free boundary, caused by the temperature dependence of the surface tension coefficient. For temperature, one of the following conditions is used: the Dirichlet condition or the homogeneous Neumann condition.

The equations under consideration contain a single dimensionless parameter: the ratio of the relaxation viscosity coefficient to the square of the Marangoni boundary layer thickness. As this parameter tends to zero, the equations transform into the Marangoni boundary layer equations for a Newtonian fluid. A characteristic feature of these equations is the dependence of the pressure on the transverse coordinate. Another feature is the presence of a time derivative in the dynamic boundary condition on the free surface.

A group analysis of the resulting system of equations was performed, and its invariant solutions were studied. Among these, one case describes unsteady motion near a stagnation point on a free boundary. The influence of relaxation viscosity on fluid flow regimes in the vicinity of the stagnation point is studied.

Another class of boundary layer problems for second grade fluids concerns fluid flow near a solid impermeable wall at high Reynolds numbers. It turns out that the resulting equations exactly coincide with the Marangoni boundary layer equations. A self-similar solution is constructed for isothermal flows.

Using the differential constraint method [3], an exact solution to the equations of a plane unsteady Prandtl boundary layer with large functional arbitrariness was constructed.

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The Importance of Breaking Symmetries

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Symmetry is essentially an attribute of objects that was studied by the Greeks as part of Geometry. It became a mathematical tool for solving algebraic and differential equations. It entered Physics with its use for understanding conservation laws. Till this stage it was the exact symmetry that was all-important, even though approximate symmetry was given some consideration. But with its use in High Energy Physics it is the breaking of the symmetry that is crucial. In this talk I try to review some aspects of approximate symmetries and of the breaking of symmetry that have assumed special significance.

Existence and uniqueness of C^1 Solution to the Goursat Problem for the dusty gas flow

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This work investigates the interaction of backward and forward rarefaction waves for the 1-D isentropic dusty gas flow. Mathematically, this type of problem involves Goursat boundary value problem. By employing characteristic analysis and assuming the absence of vacuum in the initial data, it has been established that the vacuum does not appear within the interaction domain; indeed, a vacuum will appear if the interaction time is large enough. By employing a priori estimates, we have established the existence and uniqueness of C^1 solution to the Goursat problem on the interaction domain.

Keywords. Euler System; Dusty gas; Riemann problem; Three piecewise constant data; Interaction of Rarefaction waves; Vacuum.

Note: This paper has also been published in "Journal of Mathematical Physics".

Publication Detail: Radhanandan Mandal, and M. Zafar. *Existence and uniqueness of C^1 solution to the Goursat problem for the dusty gas flow.* ***Journal of Mathematical Physics*** (**AIP**) (2025), DOI: https://doi.org/10.1063/5.0203195.

Spline Based Computational Technique for Singularly Perturbed Fredholm Integro-Differential Problems

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In this work, we develop a computational method for singularly perturbed Fredholm integro-differential equations using a spline based discretization. The scheme addresses the challenges of the singular perturbation parameter ϵ through a tension and compression spline technique, coupled with Simpson's rule for quadrature approximations. We analyze the stability and convergence properties of the proposed algorithm. Through the computation of maximum absolute errors on varying mesh sizes, we demonstrate the method's effectiveness. Numerical results indicate that the scheme yields accurate solutions and exhibits a consistent rate of convergence for arbitrarily small values of ϵ.

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Generalized viscosity implicit scheme with Meir-Keeler contraction for inclusion and fixed point problems in Banach spaces

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In this paper, we address two interconnected problems: (i) a variational inclusion problem involving an m-accretive mapping together with a finite collection of inverse strongly accretive mappings, and (ii) a fixed point problem concerning an infinite family of strict pseudo-contractive mappings in Banach spaces. To approximate their common solution, we develop a generalized viscosity-type implicit iterative scheme based on Meir–Keeler contractions. We establish a strong convergence theorem for the proposed scheme. Furthermore, we illustrate its utility by deriving applications to convex minimization, linear inverse problems, variational inequalities, and equilibrium problems. The practical relevance of the result and its applications is demonstrated through three numerical examples. Our findings extend, unify, and improve several existing results available in the literature.

Exact Solutions for Nonlinear Partial Differential Equations using Invariant Subspace Method

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Non Linear Partial differential equations (NLPDEs) are fundamental tools for modelling physical, biological, and Engineering phenomena and obtaining the exact solutions of PDEs play an important role in both theory and practice. Exact solutions often provide clear insight into the underlying physics of a problem such as system behaviour, diffusion rates, wave speeds, or stability conditions. The Invariant Subspace Method (ISM) is one of the powerful analytical techniques to obtain exact solutions of nonlinear partial differential equations. It is particularly effective for constructing finite-dimensional exact solutions by reducing a PDE to a system of ordinary differential equations (ODEs). In this talk, an essence of ISM is revealed and the method of obtaining solution is explained in great detail. Several examples are illustrated for a better understanding of ISM and its varieties of Invariant subspaces. The later part of the talk presents a research problem wherein the derivation of an exact solution to a nonlinear third-order fractional partial differential equation known as Padé-II equation is considered. It describes the unidirectional propagation of long waves in dispersive media with memory-dependent effects. The solution is based on the Invariant Subspace Method, extended to incorporate Caputo-type fractional derivatives. A valid subspace is derived based on Invariant conditions and an exact analytical solutions are derived for the fractional Padé-II equation. Its merits and limitations are also discussed.

Fuzzy Finite Element Method for Uncertainty Modelling

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Fuzzy numbers are used in the Fuzzy Finite Element Method (FFEM) to represent input uncertainties like stresses and material characteristics. A set of acceptable values limited by its membership function is defined by each fuzzy parameter. FFEM ensures deterministic coverage of uncertainty by adhering to the set-based uncertainty concept that was put forth by Lotfi A. Zadeh. Without making any assumptions about probability distributions, the aim of FFEM is to assess the structural response for any possible combination of these functional inputs. Through the decomposition of fuzzy parameters into α-level intervals and interval optimization over the resulting Cartesian product of admissible input domains, the Fuzzy Finite Element Method systematically evaluates all admissible combinations of uncertainty, ensuring comprehensive coverage of physically possible system configurations. ϵ.

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Common Fixed Point Results for Multi-Fuzzy Mappings in Generalized Metric Spaces

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FFixed point theory in MR-metric spaces has recently gained attention due to its ability to model complex interactions beyond classical metric settings. Although MR-metric spaces provide a flexible generalization of classical metric spaces and have recently been employed to study fixed points of fuzzy mappings, existing results are largely restricted to single mappings or pairs of fuzzy mappings. In practical applications such as decision-making, optimization, and uncertainty modeling, systems are often governed by multiple interacting fuzzy mappings. However, the current literature on fuzzy mappings in MR-metric spaces primarily addresses single-valued or dual-mapping cases, leaving multi-mapping interactions unexplored. This limitation poses a significant challenge when modeling systems governed by several fuzzy operators acting cyclically. Consequently, the problem addressed in this study is to investigate the existence and uniqueness of common fixed points for four and six fuzzy mappings in complete MR-metric spaces using Hausdorff MR-contractive conditions.

Keywords: Fixed point theory, MR-metric spaces, common fixed points, existence, uniqueness

Introduction

This study investigates the convergence properties and fixed-point theory of self-mappings in MR-metric spaces, which represent a modern generalization of classical metric spaces. The work establishes fundamental theorems concerning the existence and uniqueness of fixed points for contraction-type mappings, examines the behavior of Cauchy sequences, and analyzes the convergence of iterative processes toward fixed points. These results have significant implications in fields such as optimization, machine learning, and numerical analysis, and they provide a rigorous theoretical foundation for further research in the context of MR-metric spaces.

The present work extends the main results of [1] paper by generalizing the common fixed point framework from two or three fuzzy mappings to four and six fuzzy mappings in complete MR-metric spaces[2]. The approach begins by adopting the structure and contractive conditions[1] used in the original theorem and reformulating them in terms of Hausdorff MR-distances for multiple mappings. Appropriate compatibility or commutativity conditions among the mappings will be introduced to ensure the convergence of the iterative sequences generated by these mappings.By constructing suitable sequences and applying the Hausdorff MR-contractive condition repeatedly, we aim to prove that the sequences are Cauchy and converge in the complete MR-metric space. The limit point will then be shown to be a common fixed point of all the considered fuzzy mappings. Finally, uniqueness will be established by demonstrating that any two such fixed points must coincide under the given contractive condition. This methodology closely follows the logical structure of the original theorem while extending it to higher numbers of fuzzy mappings

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Differential Equation Based Modeling of Vibrational Behavior in Material-Graded Perforated Nanobeams

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This work develops a mathematical model for the vibration analysis of axially graded perforated nanobeams on elastic foundations. The formulation of the governing differential equation is derived based on the Euler–Bernoulli beam theory with nonlocal elasticity. The objective of this study is to examine the effects of material gradation and perforations on the vibrational behavior of nanobeams by solving the governing differential equation, with emphasis on how variations in perforation geometry and material gradation influence mode shapes and natural frequencies. The modified equivalent models are employed to capture the periodic perforations effect. The Galerkin method is then applied to solve the governing equations and obtain the natural frequencies and corresponding mode shapes. The applied numerical scheme gives high accuracy, as verified by comparison with existing results from the literature. Perforation and material gradation significantly impact the vibrational characteristics, governed by the nonlocal effects and geometric configuration. Numerical findings show that the dynamic response strongly depends on the filling ratio, perforation geometry, and foundation effects under different boundary conditions.

Nonlinear Lie–Hamilton systems

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Joint work with J. de Lucas (Warsaw University) and F. J. Herranz (Univ. Burgos, Spain).

Lie–Hamilton systems constitute an interesting class of Lie systems, i.e., t-dependent systems of first-order ordinary differential equations admitting a generally nonlinear superposition rule [1, 2], with enriched algebraic properties through the compatibility with respect to a Poisson structure, hence providing more types of ODE systems and the combination with additional geometric and analytical tools, such as the Lie symmetry analysis [3].

The aim of the talk is to report on a further generalization of LH systems recently proposed [4]: the so-called nonlinear Lie–Hamilton systems. In short, a nonlinear LH system is a t-dependent Hamiltonian system on a Poisson manifold (M, Λ) related to a t-dependent Hamiltonian function $h(t, x) = F(t, h_1, \ldots, h_r)$, where $h_1, \ldots, h_r$ span an r-dimensional Lie algebra of functions relative to the Poisson bracket of the Poisson manifold and F is any t-dependent function. These dynamical systems can further be interpreted as a t-dependent function of the form $h(t, x) = F(t, J(x))$ for an appropriate function $F \in C^\infty(\mathbb{R} \times \mathfrak{g}^*)$ and a momentum map $J : M \to \mathfrak{g}^*$. As application, a t-dependent generalization of the Hénon–Heiles system is considered.

This work was partially supported by the research grants PID2023-148373NB-I00 funded by MCIN/AEI/10.13039/501100011033/FEDER, UE and PID2024-156578NB-I00 funded by MICIU /AEI /10.13039/501100011033/FEDER, EU.

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Metric and topological properties of solution sets of systems of ordinary differential equations with effects specified as functional parameters

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The differential equations of mathematical models of technical systems containing parameters changing in certain ranges are considered. Such parameters include frequencies and amplitudes of external forces, environmental effects, and other quantities that can be characterized either as disturbing effects or as control actions. In many problems, only the boundaries of the values of the action parameters are known, which leads to the appearance of solution sets of differential equations.

To determine the topological and metric characteristics of solution sets, the effects of the system are considered as functional parameters of the right-hand sides of ODE systems. This helps to effectively evaluate the topological properties of solution sets and their boundaries, as well as the metric properties of solution sets. The topology of solution sets is specified by continuous mappings of solutions onto solution spaces, in which close points of the definition domain go to close points of the value range. The metric properties of solution sets are defined as properties of spaces that remain unchanged with continuous changes in the shape and size of solution domains, for example, the properties of connectivity and compactness.

For the problem with perturbing action, the set of all possible solutions (trajectories) will be written as follows:

$$Y(t, Y_0) = \left\{ y(t, y_0) : \forall y(t_0)) \in Y_0, \forall u(t) \in U, \forall t \geq 0, \frac{dy}{dt} = f(t, y(t), u(t)) \right\}.$$

The report describes new results of using symbolic-numerical methods [1]-[4] and topological and metric properties of solution sets for estimating solution sets of ODE's. The nonlinear formulas of variation of constants applied in the paper describe the relationship between the solution $y(t)$ of system ODE and the solution $z(t)$ of the perturbed system.

$$z(t) = y(t) + \int_{t_0}^{t} \frac{\partial y}{\partial y_0}(t, z, z(s)) \cdot g(s, z(s)) ds. \quad (1)$$

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On the closure of moment chains in the kinetic model of cold plasma

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Closing moment chains in the transition from kinetic to hydrodynamic models of continuous media is a classical, very complex problem. We consider it for the electron plasma model in the simplest one-dimensional case for which analytical results exist. In the kinetic formulation, the model was written by Landau [1]. The Cauchy problem for the kinetic model was investigated by Iordanskii, and it was found, in particular, that the solution preserves global smoothness for smooth initial data [2]. However, the hydrodynamic analogue of this problem has completely different properties, namely, it is known that there is a wide class of smooth initial conditions for which smoothness is lost in a finite time [3]. The hydrodynamic model of cold plasma is the result of closing moment chains at the first step.

We study the phenomenon of loss of smoothness properties of the Cauchy problem solution upon transition from the kinetic model of Landau cold plasma to a hydrodynamic model. To do this, we use a new method for closing the moment chain, yielding at each step a hyperbolic system that can be interpreted as a system describing a multiphase medium in which particles of each phase transform into one another under the influence of an electric field. The chain is closed by assuming that the last phase is pressureless.

Such a moment chain cannot be truncated; at each step, all higher moments are expressed in terms of lower moments according to a certain rule following from the condition of uniquely reconstructing the distribution over its moments.

For the special case of traveling waves, we show that for initial data of general form, already at the second step of closure we obtain a system with a globally smooth solution in time.

The study is supported by RSF grant 23-11-00056 through RUDN University.

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Secure control design for of complex partial differential systems subject to actuator fault

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This work investigate synchronization problem of N-coupled complex partial differential systems (PDSs) with mixed time-varying delay. In order to attain the synchronization, an effective fault-tolerant memory feedback control strategy is designed to guarantee the synchronization by incorporating the cyber-attacks. Bernoulli-distributed random variables are employed to indicate the success rate of cyber-attacks and occurrence of random uncertainty. By designing suitable set of Lyapunov-Krasovskii functional using novel techniques such as Kronecker product, Jensen's inequality, and S-procedure, a new set of required condition is framed in form of linear matrix inequality, to guarantee the proposed model is asymptotically synchronized. Finally, efficacy of the attained analytical results are illustrated by numerical simulation.

Keywords: Partial differential systems, distributed time-varying delay, actuator faults, cyber-attacks, memory feedback control.

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An Effective Numerical Approach to Option Pricing with an Underlying Asset

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A high-order compact finite difference scheme on a uniform mesh is proposed for solving the time-fractional Black–Scholes partial differential equation governing European-type options. The time-fractional derivative is discretized using the $L2-1_\sigma$ formula, resulting in an overall accuracy of $O((\Delta t)^2 + (\Delta x)^4)$. A rigorous stability and convergence analysis of the proposed scheme is presented. Numerical experiments validate the theoretical results and demonstrate the superior accuracy and computational efficiency of the proposed method compared with existing schemes.

The authors gratefully acknowledge financial support by UGC grant comission with NTA Ref. No. 211610100146.

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A Numerical Scheme Based on Fractional Lagrange Polynomials for Nonlinear Fractional Volterra–Fredholm Integro-Differential Equations

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This work presents an efficient numerical technique based on fractional-order Lagrange polynomials for solving a class of nonlinear fractional Volterra–Fredholm integro-differential equations. The fractional derivative is considered in the Caputo sense. The existence and uniqueness of the continuous solution are established. By employing the Laplace transform, operational matrices corresponding to fractional integration of fractional-order Lagrange polynomials are derived. These matrices reduce the original problem to a system of algebraic equations, significantly simplifying the computational procedure. An error analysis is carried out, and an upper bound for the approximation error in the L^2-norm is obtained. Numerical experiments demonstrate that the proposed method achieves high accuracy and rapid convergence, with the error decreasing sharply as the number of basis functions increases, thereby confirming the effectiveness and reliability of the approach.

Apply Conservation laws to calculate the voltage of an Aircraft skin element

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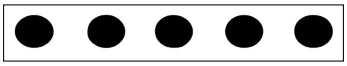
It shows a plate made of elastic material held together by elastic strings. The cross-section of this plate has the form



The system of equations for the stressed state of the plate has the form

$$\partial_x \sigma_x + \partial_y \tau = 0, \partial_y \sigma_y + \partial_x \tau = 0, \Delta(\sigma_x + \sigma_y) = 0. \tag{1}$$

The boundary conditions on the outer contour have the form

$$\sigma_x n_1 + \tau n_2 = X, \tau n_1 + \sigma_y n_2 = Y, \tag{2}$$

where $n1, n2$ are the components of the vector of the external normal to the external contour, X, Y are the specified vector; on the surface of the i-th fiber, the conditions are fulfilled

$$\sigma_x {n^i}_1 + \tau {n^i}_2 = 0, \tau {n^i}_1 + \sigma_y {n^i}_2 = 0, \tau^i = \tau_0. \tag{3}$$

Here ${n^i}_1, {n^i}_2$ are the normal vector to the i-th fiber, r_0 is a constant.
Problem (1) - (3) is solved in two stages; at the first stage, researchers construct conservation laws for the boundary value problem for the harmonic function $\sigma_x + \sigma_y$, at the third stage, researchers construct conservation laws for solving the boundary value problem for the remaining components of the stress tensor.
As a result, analytical expressions are constructed to determine the components of all components of the stress tensor at any point that is bounded by the outer contour and contours of the reinforcing fibers.

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Mathematical Analysis of Brain Tumors

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This work reviews certain combined approach which were introduced by using multi-scale (3-dimensional) CNNs along with K-means and spatial fuzzy C-means clustering to create a fully automatic brain tumor segmentation and classification scheme for MRI scans[1, 2]. The model performs analysis of each of the multi-view images at multiple spatial resolutions, eliminating the need to first apply any preprocessing step. This is accomplished with inspiration from the structure of the human visual system and allows the model to address many of the problems associated with MRI scans as well as to improve the performance of the model by incorporating an efficient initial partitioning method, and enhancing the fuzzy boundaries through fuzzy boundary enhancement techniques. This approach proved to be more beneficial as compared to both traditional machine learning algorithms and previous deep learning models.

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Memory-Driven Epidemic Dynamics with Environmental Transmission and Vaccination Incentives

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This work presents a fractional-order SIS epidemic model incorporating vaccination, incentive-based control strategies, and environmentally mediated transmission under seasonal variability. The model is formulated using the Caputo–Fabrizio fractional derivative [1], which captures memory effects without singular kernels and is well-suited for realistic disease dynamics. Transmission occurs through both direct human contact and a contaminated environment, with infectivity varying seasonally. Public health interventions such as vaccination coverage, sanitation efforts, awareness programs, and government incentives are explicitly included as control parameters [2]. Analytical results include the derivation of the basic reproduction number and conditions for disease persistence and elimination. Numerical simulations illustrate the influence of fractional order and control measures on long-term disease behavior, highlighting the stabilizing role of memory effects [3]. The proposed framework demonstrates how fractional dynamics can enhance epidemic control strategies and provides insights relevant to public health planning for recurrent, environmentally driven outbreaks.

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Existence, Uniqueness & Stability analysis of impulsive implicit neutral delay systems involving tempered Caputo fractional derivatives

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A class of impulsive neutral fractional functional differential systems involving the tempered Caputo fractional derivative is examined in this study. A neutral term with state dependency, memory effects via functional arguments, nonlinear perturbations, and external control inputs are all included in the model. The system has a history-dependent starting function and is sensitive to finite impulsive effects characterized by state-dependent impulse operators. The idea of a mild solution is developed by using appropriate solution operators related to the tempered fractional derivative. In a suitable Banach space context, fixed-point techniques are used to establish existence and uniqueness results. Additionally, sufficient requirements guaranteeing different stability qualities are derived, such as generalized Ulam-Hyers stability and Ulam-Hyers stability. The impact of the tempering parameter and fractional order on the system dynamics is highlighted by the explicit limitations for the stability constants that are established. A strong analytical basis for controlled fractional systems with memory and impulsive behavior is provided by the theoretical results, which also expand and unify a number of current models of fractional differential equations with delay and impulses.

Fractional system

$$\begin{cases} {}_0^{\lambda,C}D_t^{\alpha}\big[x(t) - N(t,x(t))\big] = Ax(t) + Bu(t) + f\Big(t, x_t, {}_0^{\lambda,C}D_t^{\alpha}x(t), u(t)\Big), \\ \qquad\qquad\qquad\qquad t \in [0,\top] = \mathcal{J}, \quad 0 < \alpha \le 1,\ \lambda \ge 0, \\ \Delta(x)\Big|_{t=t_i} = I_i(x_{t_i^-}), \quad i = 1,2,\dots,n, \\ x(t) = \varphi(t), \quad t \in [-\mathfrak{r}, 0], \\ x(0) = x_0. \end{cases} \tag{1}$$

Explanation of the problem

- ${}_0^{\lambda,C}D_t^{\alpha}$ denotes the *tempered Caputo fractional derivative* of order $\alpha \in (0,1]$ and tempering parameter $\lambda \ge 0$.
- Let $(\mathbb{X}, \|\cdot\|)$ be a real Banach space and $A : D(A) \subset \mathbb{X} \to \mathbb{X}$ be the infinitesimal generator of a C_0-semigroup $\{T(t)\}_{t\ge 0}$ on $\mathbb{X}$.
- The functions
$$N, G : \mathcal{J} \times \mathbb{X} \to \mathbb{X}, \qquad f : \mathcal{J} \times \mathbb{X}^3 \to \mathbb{X}$$
are assumed to be continuous.
- $0 = t_0 < t_1 < t_2 < \cdots < t_n = \top$ and $I_i : \mathbb{X} \to \mathbb{X}$ are continuous impulse operators.
- The jump of the state at $t = t_i$ is defined by
$$\Delta x\Big|_{t=t_i} = x(t_i^+) - s(t_i^-),$$

where

$$x(t_i^-) = \lim_{\varepsilon \to 0^-} x(t_i + \varepsilon), \qquad x(t_i^+) = \lim_{\varepsilon \to 0^+} x(t_i + \varepsilon).$$

- For any $t \in \mathcal{J}$, define the history segment $x_t : [-\mathfrak{r}, 0] \to \mathbb{X}$ by

$$x_t(\theta) = s(t + \theta), \qquad -\mathfrak{r} \leq \theta \leq 0.$$

- Solution space is defined by

$$\Omega = \mathcal{PC}([-\mathfrak{r}, \top], \mathbb{X})$$

denote the Banach space of $\mathbb{X}$-valued piecewise continuous functions endowed with the norm

$$\|x\|_\Omega = \sup_{t \in [-\mathfrak{r}, \top]} \|x(t)\|_{\mathbb{X}}.$$

Highlights and Methodology

- A new class of *implicit impulsive neutral functional differential equations* with the *tempered Caputo fractional derivative* is investigated for the solution existence.
- The model incorporates memory-dependent non-linearities, delay effects, and external control inputs.
- The model deals *state-dependent impulsive effects*, *memory-dependent nonlinearities*, and *delay terms*.
- Existence and uniqueness of *mild solutions* are established using *fixed-point techniques*, such as Krasnoselskii fixed point theorem and Banach contraction principle.
- *Ulam–Hyers* and *generalized Ulam–Hyers* stability results are derived to verify the quality of the mild solution.
- The influence of fractional order and tempering parameter on system dynamics is analyzed.

Financial support - Not Applicable.

Soliton dynamics in media with positive dispersion

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Soliton formation and interaction in the non-integrable rotation-modified Korteweg de Vries (rmKdV) and rotation-modified Benjamin–Ono (rmBO) equations with "anomalous" (positive) dispersion are considered. These equations are applicable to the description of wave processes in plasma, shallow liquid films, and possibly in other media. It is shown that specific solitons with zero total mass and non-monotonic asymptotics can emerge from pulse-type initial perturbations of a certain polarity. These solitons can form either regular trains ranked by amplitude, or irregular nonstationary configurations of bounded interacting solitons, or stationary moving multi-solitons. Through the numerical modelling it is demonstrated that the interactions of solitons in the rmKdV and rmBO equations are inelastic, resulting in the creation of a "soliton-champion" within closed systems. For instance, in systems with periodic boundary conditions, only the soliton with the greatest amplitude persists, effectively eliminating all other solitons after interacting with them. Such solitons are rather robust; they gradually decay under the influence of weak dissipation, ordinarily radiating small-amplitude quasi-linear wave trains.

Dynamics of the Conformation Tensor Invariants in Models with Quadratic Nonlinearity

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The study is devoted to the analysis of the dynamics of the conformation tensor in rheological models of polymer solutions with quadratic nonlinearity. To ensure the invariance of equations with respect to rotations and shifts, the apparatus of the Lie derivative is used, replacing the classical material derivative. This approach corresponds to the principle of material objectivity, fundamental for rheologically complex media.

The equation for the conformation tensor $\mathbf{C}$ with a quadratic right-hand side and constant coefficients is considered. By combining this equation with the Hamilton–Cayley identity, the problem is reduced to a closed system of ordinary differential equations for the tensor invariants along fluid trajectories.

Through a logarithmic derivative substitution and a series of computations, the nonlinear system is reduced to a linear differential equation with constant coefficients, which allows obtaining the general solution in explicit form.

The obtained results enable the analysis of the qualitative behavior of polymer structures in the flow, including stretching, compression, and relaxation regimes. The method demonstrates the effectiveness of combining geometric and algebraic approaches for studying nonlinear rheological models.

Some linear models were studied by the authors in previous works (see [3] and [4]).

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On the solutions of the Cauchy problem for generalized stochastic and deterministic Korteweg-de Vries equations

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Let *noise* be a random process $V(t)$, $V(0) = 0$, with continuous realizations with probability 1; in particular, this could be a Wiener process or fractal Brownian motion. Since the technique used in this paper is path-based, we can take as noise an arbitrary continuous, possibly non-differentiable function $V(t)$, $V(0) = 0$. The Cauchy problem for the generalized Korteweg-de Vries (GKdV) equation is:

$$u_t + (f(u))_x + u_{xxx} = 0, \quad u(x, 0) = u_0(x), \; x \in R.$$

Let us consider the Cauchy problem for the stochastic generalized Korteweg-de Vries (SGKdV) equation with noise in the nonlinear term in the form of a symmetric integral [1], where $u(x,t) = u(x,t,V(t))$:

$$d(u)_t + (f(u))_x * dV(t) + u_{xxx}dt = 0, \quad u(x, 0, V(0)) = u_0(x). \tag{1}$$

The differential with a symmetric integral is written as: $(u)_t dt = u_t dt + u_v * dV(t)$, therefore, the SGKdV takes the form:

$$[u_t + u_{xxx}]dt + [u_v + (f(u))_x] * dV(t) = 0. \tag{2}$$

The solution of the Cauchy problem for (2) is represented as a function of three variables $u(x,t,V(t))$, $u(x,0,V(0)) = u_0(x)$, where $u(x,t,v)$ is a sufficiently smooth function whose differential with a symmetric integral satisfies equation (2). Via $Ai(z) = \frac{1}{\pi}\int\limits_0^{\infty} \cos\left(\frac{y^3}{3} + yz\right) dy$ will denote the Airy function of the first kind. The following result was obtained for this problem [2].

Theorem. *Let the function*

$$u(x,t,V(t)) = (3t)^{-\frac{1}{3}} \int\limits_{-\infty}^{+\infty} Ai\left(\frac{x-y}{(3t)^{\frac{1}{3}}}\right) u(y,0,V(t))\, dy \tag{3}$$

where $u(y,0,V(t))$ is the solution to the Cauchy problem for the equation

$$\frac{\partial}{\partial v} u(x,t,v)|_{v=V(t)} + f'(u(x,t,V(t))) \frac{\partial}{\partial x} u(x,t,V(t)) = 0, \quad u(x,0,0) = u_0(x)$$

for $t = 0$, $f(u) \in C^2(R)$, $u_0(x) \in C^3(R)$, then the Cauchy problem for the equation (2) has an approximate solution $u(x,t,V(t))$, which is represented as relation (3).

Remark. The case when noise affects both the nonlinear and dispersion terms, cases with noise in the dispersion term or on the right-hand side of the equation are studied in [3, 4]. This report will present a numerical method and the results of modeling solutions to this problem.

The author is grateful to Professor F.S. Nasyrov for his attention to the work.

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On the use of total differentiation operators for constructing numerical algorithms for involutive systems

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This paper proposes a method for constructing numerical algorithms for solving boundary value problems for systems of differential equations. The algorithm is based on an approximate solution of Lie equations for operators of total differentiation.

The jet of any function is an invariant manifold under the operators of total differentiation.

The jet of any solution of an involutive system is an invariant manifold under the projections of the operators of total differentiation onto the manifold of the equation [1].

The boundary conditions are a submanifold of the solution. And the result of the action of the approximate group on the boundary conditions will belong to the manifold of the approximate solution of the boundary value problem. And with an appropriate choice of operator from the ideal of operators of total differentiation, it will yield the entire manifold of the solution.

For example, for the Euler equations

$$\begin{aligned} u_t + uu_x + vu_y + p_x/\rho &= 0, \\ v_t + uv_x + vv_y + p_y/\rho &= g, \\ u_x + v_y &= 0 \end{aligned} \tag{1}$$

projections of the operators of total differentiation onto the manifold of the system 1 have the form

$$\begin{aligned} D_x &= \partial_x + u_x\partial_u + v_x\partial_v + p_x\partial_p + \cdots, \\ D_y &= \partial_y + u_y\partial_u - u_x\partial_v + p_y\partial_p + \cdots, \\ D_t &= \partial_t - (uu_x + vu_y + p_x/\rho)\partial_u - (uv_x - vu_x + p_y/\rho - g)\partial_v + p_t\partial_p + \cdots \end{aligned}$$

Using the following operator allows us to construct a numerical algorithm with a variable grid.

$$D_t + \xi_1(x, y, t)D_x + \xi_2(x, y, t)D_y.$$

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Spatially Biased Particle Dynamics in Coupled Transport Processes

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In the real world, transport processes are actively present in various disciplines. Motivated by a real-life flyover traffic scenario, we proposed a two-lane traffic system where the particle dynamics are biased based on the segments of the lane. To understand the role of a flyover in the traffic system, we consider a coupled one-dimensional lattice in which the lattice is divided into three equal segments. On that, particles obey the lane-switching behavior in the middle part of the lane, and other parts of the lane obey the particle attachment and detachment dynamics. The overall behavior of the systems has been analyzed by employing the hybrid mean-field approximation technique. The coupled system of partial differential equations is solved by utilizing the numerical scheme of finite difference method, and the obtained numerical results are verified through Monte Carlo simulation. It is observed that the system exhibits a greater number of stationary phases when particles follow the asymmetric coupling behavior, and more interestingly, it is found that due to the high possibility of particle inclusion rather than exclusion, the system experiences a unique double shock phase.

Similarity reductions and invariant solutions of $(3+1)$–dimensional generalized Konopelchenko-Dubrovsky-Kaup-Kupershmidt model via Lie symmetry analysis

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This work is devoted to the analytical study of the (3+1)–dimensional generalized Konopelchenko-Dubrovsky-Kaup-Kupershmidt (gKDKK) model, which describes nonlinear wave phenomena in fluid mechanics, plasma physics, and ocean dynamics. The Lie symmetry method is first applied to obtain similarity transformations, resulting in a six-dimensional Lie algebra and its associated commutation relations. Based on these symmetries, several similarity reductions and invariant solutions are derived. Additionally, traveling wave solutions of physical relevance are obtained using the improved F- expansion method. Finally, the dynamical behavior of different solution structures including one-soliton, two-soliton, three-soliton, multisoliton, peakon-type soliton, wave-like, and parabolic-type solutions is investigated, and the corresponding solution profiles are illustrated graphically.

A Study on Existence, Uniqueness and Ulam–Hyers Stability of Impulsive Neutral Hilfer Fractional Control Systems with Nonlocal Conditions

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This paper investigates a class of impulsive neutral control systems governed by Hilfer fractional derivatives with nonlocal initial conditions. The proposed model incorporates neutral terms, impulsive effects, and nonlinear integral functional, thereby capturing memory properties, hereditary effects, and instantaneous state variations. By employing the theory of strongly continuous semigroups, an equivalent formulation of mild solutions is derived.The existence of at least one mild solution is established via Sadovskiis fixed point theorem by proving that the associated solution operator is condensing.Uniqueness of the mild solution is obtained using a suitable fractional Gronwall inequality under appropriate Lipschitz-type assumptions. Furthermore, the stability behavior of the system is examined in the sense of Ulam - Hyers (UH) stability and Generalized-Ulam - Hyers (GUH) stability, ensuring continuous dependence of solutions on small perturbations. An illustrative example is presented to support the theoretical results. The obtained results extend and unify several existing studies on impulsive fractional differential systems by incorporating Hilfer-type derivatives, neutral structures, and integral non - linearities within a unified framework.

Proposed System

Consider the impulsive neutral fractional control system:

$$\begin{cases} {}^{\mathcal{H}}_{0}D^{\alpha,\kappa}_{t}\big[x(t)-\mathcal{N}(t,x(t))\big] = Ax(t)+Bu(t)+f\big(t,x(t),{}^{\mathcal{H}}_{0}D^{\alpha,\kappa}_{t}x(t),G(t)\big), & t\in\mathbb{J}:=[0,T],\\ x(t_m^+)-x(t_m^-)=\mathcal{I}_m\big(x(t_m^-)\big), & m=1,2,\ldots,n,\\ I^{1-\eta}_{0^+}x(0)=\delta-\sum_{j=1}^{k}C_j\,x(t_j), & \\ G(t)=\int_0^T g\big(t,s,x(s)\big)\,ds. & \end{cases} \tag{1}$$

The proposed system (1) involves the Hilfer fractional derivative of non-integer order $0<\alpha<1$ and type $\kappa\in[0,1]$, denoted by ${}^{\mathcal{H}}_{0}D^{\alpha,\kappa}_{t}$. The operator $A: D(A)\subset\Omega\to\Omega$ is a closed, densely defined linear operator that generates a strongly continuous semigroup on Ω. The control function $u(t)$ belongs to $L^2(\mathbb{J},\Omega)$ and the neutral term $\mathcal{N}$ is also continuous.The nonlinear function $f:\mathbb{J}\times\Omega^3\to\Omega$ is continuous on $\mathbb{J}$ and depends on the current state, the fractional derivative, and the integral memory term $G(t)$.The fractional integral $I^{1-\eta}_{0^+}$ of order $1-\eta=(1-\alpha)(1-\kappa)$ defines the nonlocal initial condition. The term $\sum_{j=1}^{k}C_jx(t_j)$ represents a multi-point nonlocal condition, where C_j are constants.For $t\in(-r,0]$, $x(t)$ is continuous and satisfies $x(t_j)=x(t_{j+s})$ for $-r\le s<0$.The impulsive condition models instantaneous state jumps at discrete moments t_m, where the jump magnitude is governed by the impulse operator $\mathcal{I}_m$ and depends on the state immediately before the impulse.

Explanation of the problem

- **Hilfer fractional derivative:** ${}_0^{\mathcal{H}}D_t^{\alpha,\kappa}$ of order $0 < \alpha < 1$ and type $\kappa \in [0, 1]$ captures *memory* and *hereditary effects* inherent in the system.
- **Neutral term:** $x(t) - \mathcal{N}(t, x(t))$ indicates that the derivative depends not only on the current state $x(t)$ but also on its delayed or modified value $\mathcal{N}(t, x(t))$, modeling internal feedback or delay effects.
- **Linear operator:** $Ax(t)$ represents the infinitesimal generator of a strongly continuous semigroup, describing the linear dynamics of the system.
- **Control input:** $Bu(t)$ is an external control term allowing regulation of the system behavior.
- **Nonlinear term:** $f\big(t, x(t), {}_0^{\mathcal{H}}D_t^{\alpha,\kappa}x(t), G(t)\big)$ represents nonlinear interactions depending on the state, fractional derivative, and a nonlocal integral functional $G(t)$.
- **Impulse effects:** At discrete times t_m, the state experiences instantaneous jumps modeled by $\mathcal{I}_m(x(t_m^-))$, capturing shocks, switches, or resets.
- **Nonlocal initial condition:** $I_{0^+}^{1-\eta}x(0) = \delta - \sum C_j x(t_j)$ relates the initial state to multiple points in the past, reflecting history-dependent processes.
- **Integral functional:** $G(t) = \int_0^T g(t, s, x(s))\, ds$ accounts for cumulative or distributed effects over the interval $[0, T]$, modeling memory, feedback, or long-term interactions.

Materials and Approaches

- A new class of nonlinear implicit impulsive fractional control systems with Hilfer fractional derivatives is investigated.
- The obtained theoretical results generalize several existing models by simultaneously incorporating Hilfer fractional derivatives, neutral terms, impulses, and nonlocal conditions.
- The system is formulated in an abstract Banach space, allowing treatment of infinite-dimensional dynamics.
- The impulsive effects depend on both current and historical states, capturing memory-driven jump behavior.
- The proposed framework is suitable for modeling complex control processes with hereditary characteristics.
- The existence of at least one mild solution for the proposed impulsive neutral Hilfer fractional system is established by applying *Sadovskiis fixed point theorem.*The associated solution operator is shown to be condensing by decomposing it into a contraction and a compact operator.
- Uniqueness of the mild solution is proved using a suitable *fractional Gronwall inequality.*Under appropriate Lipschitz-type conditions on the nonlinear and impulsive terms, the difference of two solutions is shown to vanish identically on the considered interval.
- The *Ulam-Hyers (UH) stability* and *Generalized-Ulam-Hyers (GUH) stability* of the system are investigated. It is shown that every approximate solution of the system remains close to an exact solution, ensuring robustness with respect to small perturbations.

Funding

The authors received no external funding.

The truncation method for finding solutions to nonlinear equations

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We will demonstrate the application of the truncation method using the following example. Let the Cauchy problem be given for a nonlinear equation that can be formally written as

$$u_t = L(u) + V'(t)M(u), \quad u(x,0) = \varphi(x), \quad (x,t) \in R \times [0,+\infty), \tag{1}$$

here the term $L(u)$ is the linear part of the equation, the second term represents the nonlinear component

$$L(u) = \sum_{k=1}^{n} a_k \frac{\partial^k}{\partial x^k} u, \qquad M(u) = M\left(x, u, \frac{\partial}{\partial x} u, \dots \frac{\partial^m}{\partial x^m} u\right).$$

$V(t)$ is nowhere non-differentiable function. The solution to problem (1) is sought in the form $\tilde{u}(x,t) = u(x,t,v)|_{v=V(t)}$.

Theorem. *Let the function $u(x,t,v)$ satisfies the system of equations*

$$\begin{cases} u_t(x,t,v) + L(u(x,t,v))|_{v=V(t)} = 0, \\ u_v(x,t,v) + M(u(x,t,v))|_{v=V(t)} = 0 \end{cases} \tag{2}$$

with the initial condition $u(x,0,0) = \varphi(x)$, then $\tilde{u}(x,t)$ is a solution to problem (1).

Remarks. 1. Note that in the first equation of system (2), the variable v is a parameter taken at each t point $v = t$; in the second equation, the variable t plays the role of the parameter. 2. The solution to equation (1) is usually found as follows. First, using the convolution of the fundamental solution and the "initial condition" of the form $u(x,0,v) = \tilde{\varphi}(x,v)$, the solution $u(x,t,v)$ of the first equation is constructed. This solution $u(x,t,v)$ is then substituted into the second equation of the system, solving the Cauchy problem with $u(x,0,0) = \varphi(x)$. Finally, it remains to set $v = t$.

The essence of this method is that we cut the multidimensional "linear environment" of variables (x,t,v), defined by the linear part of equation (1), with the cut $v = t$, and, in a sense, apply a dressing procedure, resulting in a nonlinear equation in the variables (x,t).

Using this method, solutions to the Cauchy problem were constructed for the Burgers equation (without the Cole-Hopf substitution), the Korteweg-de-Vries equation, the nonlinear Schredinger equation, and a number of other equations. Of course, this method allows us to construct solutions not only to evolutionary equations, but also to equations with higher-order derivatives with respect to the variable t, such as generalized Klein-Gordon equations. On the other hand, using the differential hierarchy, solutions can be constructed for entire classes of higher-order nonlinear equations.

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Dirichlet Problem For Lane-Emden Type Equations Involving Several Sublinear Terms

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We establish the existence, uniqueness, and sharp bilateral pointwise estimates for positive bounded solutions to the problem

$$\begin{cases} \mathcal{L}u = \sum_{i=1}^{m} \sigma_i u^{q_i} + \sigma_0, \quad u \geq 0 & \text{in } \Omega, \\ \liminf_{x \to y} u(x) = f(y), & y \in \partial^\infty \Omega, \end{cases}$$

where $0 < q_i < 1$. Here $\mathcal{L}u = -\mathrm{div}(\mathcal{A}\nabla u)$ is a uniformly elliptic operator with bounded coefficients, σ_i is a nonnegative Radon measure on an $\mathcal{A}$-regular domain $\Omega \subset \mathbb{R}^n$ which possesses a positive Green function associated with $\mathcal{L}$, and f is a nonnegative continuous function on the boundary $\partial^\infty \Omega$.

We also obtain an analogous result for positive continuous solutions and extend to related sublinear problems with zero boundary conditions involving the fractional Laplacian $(-\Delta)^\alpha$ for $0 < \alpha < n/2$, in $\mathbb{R}^n$.

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A physics-informed neural network based method for the nonlinear Poisson-Boltzmann equation in Electrolyte–Polyelectrolyte Systems

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Physics-Informed Neural Networks (PINNs) have emerged as a powerful mesh-free approach for solving partial differential equations by embedding physical laws directly into the neural network training process via automatic differentiation. In this study, we develop a PINN-based framework to solve the nonlinear Poisson–Boltzmann (PB) equation governing electrostatic potential distribution in heterogeneous electrolyte–polyelectrolyte systems. The proposed method considers a two-region domain with distinct dielectric permittivities, formulated in Cartesian coordinates, and incorporates the governing PB equation, interface conditions, boundary conditions, and their derivatives into a unified loss function. By avoiding traditional mesh generation and linearization procedures, the approach effectively handles discontinuous material properties across interfaces. Numerical results demonstrate that the PINN model provides accurate and stable approximations for complex nonlinear systems. This work highlights the flexibility and efficiency of PINNs for modeling electrostatic phenomena in soft-matter and heterogeneous media, with promising applications in electrokinetics, colloid science, and biophysics.

Keywords: Physics informed neural networks, Poisson–Boltzmann equations, Heterogeneous electrolyte–polyelectrolyte systems, Mesh-free methods, Automatic differentiation

Vlasov – type Equations, Miln-McCree Method and Derivation of Hubble Law for Rotation Invariant Hamiltonians and Accelerated Expansion of the Universe from Minimal Action Principle

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In classical textbooks (Pauli; Fock,; Landau and Lifshitz; Dubrovin, Novikov, Fomenko; Weinberg; Vlasov), equations for fields in the Einstein and Maxwell system of equations are proposed without deducing the right parts. Here we give the derivation of the right-hand sides of the Maxwell and Einstein equations within the framework of the Vlasov-Maxwell-Einstein equations from the classical, but slightly more general principle of least action. A method of transition from kinetic equations to hydrodynamic consequences is proposed, as it was done earlier by A.A.Vlasov himself. In the case of Hamiltonian mechanics, a transition from the hydrodynamic consequences of the Liouville equation to the Hamilton-Jacobi equation is possible [1, 2]. Thus, in the non-relativistic case, Milne-McCree solutions are obtained, and its weekly-relativistic analogues and relativistic analogues [4, 5]. New definition of Hubble constant is proposed, equation for it and accelerated expanding of Universe is discussed. We get it from relativistic Einstein action without any Lambda, dark matter or additional fields [6], and also we get that curvature of Universe in the framework of FRLW-model is negative [7, 8].

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Matrix and Inverse Matrix Projective Synchronization of Chaotic Dynamical Systems

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Differential equations play a crucial role in the study of real-world problems, including epidemiology and dynamical systems. A chaotic dynamical system consists of a system of first-order differential equations and is highly sensitive to initial conditions. In this talk, we analyze matrix projective synchronization and inverse matrix projective synchronization in fractional-order chaotic systems with uncertain terms. Several sufficient conditions are derived to achieve both types of synchronization. Furthermore, a time-delay term is incorporated into the chaotic system, providing an elegant and practical application to real-world problems.

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On stochastic and deterministic generalized Burgers equations

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Let a random process $V(t)$, $t \in [0,T]$, $T \in \mathbb{R}^+$, $V(0) = 0$, be given with almost surely continuous trajectories on the probability space $(\Omega,\ \mathcal{F},\ \mathbf{P})$. The Cauchy problem for the generalized Burgers equation with noise in the nonlinear part can be represented as follows:

$$\begin{gathered}\tilde{u}_t + f'(\tilde{u})\tilde{u}_x V'(t) = \tilde{u}_{xx},\ \ \tilde{u}(x,0) = \varphi(x) \in C^1(\mathbb{R}),\\ \tilde{u} = \tilde{u}(x,t),\ \ (x,t) \in \mathbb{R}\times\mathbb{R}^+,\end{gathered} \tag{1}$$

where $f(u)$ is a nonlinear and smooth function and $V'(t)$ is a formal derivative that may not exist, for example, if $V(t)$ is a Wiener process. Therefore, in a mathematically rigorous formulation, the Cauchy problem (1) must be rewritten in integral form:

$$\tilde{u}(x,t) - \varphi(x) + \int_0^t f'(\tilde{u}(x,s))\tilde{u}_x(x,s) * dV(s) = \int_0^t \tilde{u}_{xx}(x,s)\,ds. \tag{2}$$

The integral on the left-hand side of equation 2 is a symmetric integral [1] with respect to the process $V(t)$. The symmetric integral with respect to a continuous function is a generalization of the Stratonovich stochastic integral and coincides with it in the case of a Wiener process.

It has been shown [2] that the solution of the Cauchy problem for the stochastic generalized Burgers equation (2) can be found as a function of three variables

$$\tilde{u}(x,t) = u(x,t,V(t)).$$

Put

$$u(x,t,v) = \frac{1}{2\sqrt{\pi t}}\int_{-\infty}^{\infty} g(\xi,v)\exp\frac{-(x-\xi)^2}{4t}\,d\xi, \tag{3}$$

where $g(x,v) = u(x,0,v)$. Then the following theorem holds true:

Theorem 1. *Let the function $g(\xi,t)$ is a solution to the Cauchy problem for the generalized Hopf equation*

$$g_v + f'(g)g_x = 0,\quad g(x,0) = \varphi(x).$$

Then the function $\tilde{u}(x,t) = u(x,t,V(t))$ given by (3) *is a solution to the Cauchy problem for the stochastic generalized Burgers equation* (2).

Remark. The developed method for solving the Cauchy problem for the generalized Burgers equation with noise in the nonlinear part allows solving the problem both with and without noise. In the case $V(t) = t$, the problem (1) becomes the Cauchy problem for the deterministic generalized Burgers equation

$$\tilde{u}_t + f'(\tilde{u})\tilde{u}_x = \tilde{u}_{xx},\ \ \tilde{u}(x,0) = \varphi(x),$$

whose solution is determined from the relations

$$\begin{gathered}\tilde{u}(x,t) = u(x,t,t) = \frac{1}{2\sqrt{\pi t}}\int_{-\infty}^{\infty} g(\xi,t)\exp\frac{-(x-\xi)^2}{4t}\,d\xi,\\ g_t + f'(g)g_x = 0,\quad g(x,0) = \varphi(x).\end{gathered}$$

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